OPTIMUM DESIGN SENSITIVITY OF REINFORCED CONCRETE FRAMES

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Abstract
This paper presents design optimization of a two bay one storey reinforced concrete plane frame. Direct stiffness method was used for the structural analysis of the frame. The design variables were taken as the area of steel and the cross sectional dimension of the members. The design constraints on dimensions, strength capacities and areas of reinforcement were based on the specifications of Eurocode 2 (1992). A computer program was written using MATLAB to perform the optimization process. The optimal design results in cost savings of an average of 5.1% for the beam and 3.2% for the column. The decrease in cost is due to decrease in cross section as a result of optimization. The saving in cost is lower in the column. This can be attributed to the fewer number of design variables in column optimization. Expressions for computations of span effective depth and reinforcement ratios were obtained. Furthermore beam design charts were plotted for optimum design of reinforced concrete beams. The expressions and design charts based on optimum design concept will be valuable for preliminary and low cost design of reinforced concrete structures.

Keywords: Optimization, reinforced concrete frames, design, Eurocode 2, MATLAB

1. Introduction
An optimization problem is solved by formulating the design variables for the structural frames, the objective function that needs to be minimized and the design constraints that are imposed on the system. The code requirements for safety and serviceability as well as other performance requirements constitute the constraints. Optimization problems of structural frames have been considered by several researchers. Hussainain (1992) employed second-order method to analyse and design reinforced concrete (RC) frames. The author formulated the frames using a non-linear programming technique considering ACI 318-83(1998) building code requirements for reinforced concrete. Concrete dimensions and steel areas for columns and beams were the design variables. The objective function was the sum of all the costs for each column and beam. From the study, it was shown that there was a 3.5% reduction in cost while processing time to reach an optimum solution increased by 5%.

Balling and Yao (1997) examined the viability of the assumption that optimum concrete section dimensions are insensitive to the number, diameter, as well as longitudinal distribution of the reinforcing bars. This was achieved by comparing optimum results from a multilevel method that considered the problem as a system optimization problem and a series of individual member optimization problems. From the results, a simplified method was presented and recommended as the most efficient method for the optimization of reinforced concrete frames. Rajeev and Krishnamoorthy (1998) applied a simple genetic algorithm (SGA) to the cost optimization of two-dimensional frames. The authors concluded that genetic algorithm-based methodologies provide ideal techniques when further modification such as detailing, placing of reinforcement in beams and columns and other issues related to construction are brought into optimal design model.

Bontempi, et al. (1999) presented a systematic approach to the optimal design of concrete structures using a combined genetic algorithm and fuzzy criteria. The procedure was oriented to the optimal design of concrete frames but also suitable for other kinds of structures. Camp, et al. (2003) in a study using genetic algorithm noted that the main factors affecting cost of reinforced concrete structures are amount of concrete and reinforcement required. Guerra and Kiousis (2006) carried out optimization design of multi-storey and multi-bay reinforced concrete frames and found out that the optimal design results in cost savings for 8m and 24m spans were 1% and 17% respectively.

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The design variables in optimization of reinforced concrete are width, depth of section or effective depth and longitudinal reinforcement of members as in the studies by Booz, et al.(1984), Kanagasundaram and Karihaloo (1990) and Maharrami(1993). Booz, et al.(1984) applied the German design regulations to optimize the concrete elements while Kanagasundaram and Karihaloo (1990) performed their studies based on Australian design regulations. The work presented in this study considered the optimum design sensitivity of an RC frame designed in accordance with Eurocode 2 (1992). The frame was analysed using direct stiffness method while the optimisation procedure was implemented using MATLAB optimization tool.

2. Methodology
2.1 Analysis of the Reinforced Concrete Frames
The reinforced concrete frame is taken from the Architectural Drawing of the Administrative and Lecture Block of the School of Business Studies, Federal Polytechnic, Bauchi. The frame is shown in Figure 1. The frame consists of beam sections 225mm x 350mm and column sections 225mm x 225mm. It was analyzed using the direct stiffness method while the optimization procedure was implemented using MATLAB (1999).

2.2 Optimization
The objective function to be minimized is expressed in terms of concrete volume, steel weight, form work surface area as well as their unit costs. The mathematical form of the objective function for the design of the RC frames is given by the following expressions [Camp, et al., 2003] is:

\[
F = \sum_{\text{element}} C \cdot lbh + C_s l A_s + 2C_f l (b + h)
\]

Subject to \( c_1 \leq 0, \ c_2 \leq 0, …, c_n \leq 0 \) (1)

Where F is the objective function; \( p_m, p_j, p_s \) are material properties, connection characteristics and structural characteristic respectively; \( b \) and \( h \) are cross sectional dimensions of the members; \( l \) and \( A_s \) are the length of member and area of reinforcing bars respectively; \( C_c \) is cost of concrete per unit volume; \( C_s \) is cost of steel, ties and stirrups per unit weight and \( C_f \) is the cost of form work per unit surface area.

The formulation of the objective and constraint functions and the associated notations are as follows:

Parameters: \( f_{c,k} \) and \( f_{y,k} \) are the characteristic cylinder strength of concrete and characteristic yield strength of reinforcement respectively. \( C_{\text{beam}} \) is the cost of a critical beam in the frame and \( C_{\text{column}} \) is the cost of a critical column in the frame; \( L_{\text{m}} \) is the clear Span of beam, \( L_0 \) is the Span of beam centre to centre, \( L_n \) is the clear height of column and \( L_c \) is the span of column centre to centre. The parameters \( f_{c,k} \) and \( f_{y,k} \) are taken as 25N/m² and 500N/m² respectively.

Decision variables: For the beam, \( b_b \) and \( d_b \) are width and effective depth respectively; \( A_{az1} \) and \( A_{az2} \) are the top reinforcements at left and right supports respectively; \( A_{zt1} \) and \( A_{zt2} \) are the bottom reinforcements curtailed and full respectively at mid-span and \( A_{zbc} \) is the top reinforcement at midspan. For the column, \( b_c \) and \( d_c \) are width and effective depth of column respectively and \( A_{zc} \) is the area of longitudinal reinforcement.

The cost of reinforced concrete frame elements is determined by the following expressions:

\[
C_{\text{beam}} = C_c(V_{bc} - V_{bs} - V_{c}) + C_s \gamma_s(V_{bs} + V_{c}) + C_s A_{bf}
\]

and

\[
C_{\text{column}} = C_c(V_{cc} - V_{cs} - V_{c}) + C_s \gamma_s(V_{cs} + V_{c}) + C_s A_{cf}
\]

where \( V_{bc} \) and \( V_{cc} \) are the volume of concrete in the beam and column respectively; \( V_{bs} \) and \( V_{cs} \) are volumes of longitudinal steel in the beam and column respectively; \( V_{c} \) and \( V_{t} \) are volume of stirrups in beam and column respectively; \( A_{bf} \) and \( A_{cf} \) are the surface area of framework for the beam and column respectively and \( \gamma_s \) is the unit weight of steel.

2.2.1 Objective function for the beam
The objective function for a singly reinforced beam section shown in Figure 1 was derived as follows:

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Let $V_{gb}$ be the gross volume of the beam, $x_1$ and $x_2$ be the width $b_b$ and effective depth $d$ of the beam respectively; $d_{c1}$ and $d_{c2}$ be cover to reinforcement in beam and column respectively each taken as 50mm and $x_3$, $x_4$, $x_5$ and $x_6$ be the areas of reinforcement $A_{sbt1}$, $A_{sbt2}$, $A_{gb1}$ and $A_{gb2}$ respectively. Then from Figure 1, we have the following expressions:

$$V_{gb} = b_b (d_{c1} + d_{c2}) L_b = 270000x_1+5400x_1x_2$$

$$V_{bs} = (A_{sbt1} + 0.75A_{sbt2})L_b + A_{scb}(0.84L_b - d_{c1} - d_{c2}) + (A_{gb1} + A_{gb2})(0.25L_b + \frac{d_{c1} + d_{c2}}{2})$$

$$V_v = A_v [2(b_b + d_{c1} + d_{c2}) - 8(d_{c1} + \frac{d_{c1} + d_{c2}}{2} - d_{c2})] (\frac{L_b}{2} + 1) = 9956x_1+9956x_2+776568$$

$$A_{bf} = b_b[L_b - (d_{c1} + d_{c2})] + 2(d_{c1} + d_{c2})L_b = 5350x_1x_1x_2+108000x_2+540000$$

The costs of materials were adopted from Newpro Quants Consultants, (2008) for the year 2008 case study in the Nigerian market. The rates for concrete work were as follows:

i) Cost of all rates plain concrete (for 1:2:4-19mm agg) in super structure = N24,000 per m$^3$ = N24,000x1.0$^{-9}$ per m$^3$

ii) Cost of reinforcement = N 420.00 per Kg = N 42000.00

Then cost of reinforcement per volume = C_s[Vbs+V_v]

iii) Cost of Formwork= N950 per m²= N950 x1.0E-6 per mm$^2$

Therefore, the objective function from Equation (2) is simplified as:

$$f = 24000x1.0E-9(270000x_1+5400x_1x_2+4725x_1x_2+2875x_1x_2+1462.5x_1x_2+462.5x_1+6384x_1+6384x_2+549024) + 42000x77x1.0E-9(4725x_1+4311x_2+1462.5x_2+1462.5x_2+9956x_1+9956x_2+776568) + 950x1.0E-6(5350x_1x_1x_2+108000x_2+540000)$$

### 2.2.2 Beam constraint functions

The constraints for the beam based on EC 2 specification are derived in terms of the design variables as follows:
a) Geometric Constraint
For economy the proportions of effective depth $d$ is in the range from 1.5 to 2.0 times the width of beam, $b$ (Hassanain, 1992). Mathematically, this is expressed as:

$$1.5d_b \leq d \leq 2 \quad \text{or} \quad d - 2.0d_b \leq 0$$

Therefore,

$$x(2) - 2x(1) \leq 0 \quad (9)$$

b) Flexural capacity constraint
   i) Singly reinforced rectangular section

For a singly reinforced beam, the stress block is as shown in Figure 2 (Mosley, et al., 2007).

![Figure 2: Rectangular Stress – Block](image)

From equilibrium of forces,

$$\begin{align*}
F_{cc} &= F_{st} \\
F_{cc} &= 0.567 f'_{ck} b s \\
F_{st} &= 0.87 f_{y} A_s \\
s &= \frac{0.87 f_{y} A_s}{0.567 f'_{ck} b}
\end{align*}$$

$$M_u = F_{st} z = 0.87 f_{y} A_s (d - \frac{5}{2}) = 0.87 f_{y} A_s (d - \frac{0.87 f_{y} A_s}{1.134 f'_{ck} b})$$

Where $F_{cc}$ and $F_{st}$ are the forces in the concrete and the steel respectively; $M_d$ and $M_u$ are ultimate design moment and moment resistance of the section respectively and $A_s$ is the tension reinforcement.

The ultimate design moment should be less than the moment carrying capacity of the beam. Mathematically,

$$M_d \leq M_u = 0.87 f_{y} A_s (d - \frac{0.87 f_{y} A_s}{1.134 f'_{ck} b})$$

$$M_d - 0.87 f_{yk} A_s (d - 0.87 f_{yk} A_s/1.134 f'_{ck} b) \leq 0$$

Taking $f_{ck} = 25 \text{N/m}^2$ and $f_{yk} = 500 \text{N/m}^2$

$$\Rightarrow M_d - 435 x_3 (x_2 - 12.787 x_3/x_2) \leq 0 \quad (11)$$
Where \( x_m \) is the tension reinforcement, \( x_1 \) and \( x_2 \) are the width and effective depth of the section respectively.

ii) Doubly reinforced rectangular section

Consider a rectangular section with compression reinforcement at the ultimate limit state as shown in Figure 3 (Mosley, et al. 2007).

![Figure 3: Rectangular Stress Block](image)

The compression reinforcement is

\[
A_c = \frac{M - 0.167f_{ck}bd^2}{0.87f_{y,k}(d-d)} = \frac{M - 5.01bd^2}{435(d-d)}
\]

\[
\therefore M - 4.175x_1x_2^2 - 435x_4(x_2 - 50) = 0
\] (12)

The tension reinforcement is

\[
A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{y,k}x} + \frac{M - 5.01bd^2}{435(d-d)}
\]

\[
z = 0.82d
\]

\[
\therefore X_f = \frac{0.167f_{ck}bd^2}{356.7d} + \frac{M - 5.01bd^2}{435(d-d)}
\]

\[
X_n = 0.011x_1x_2^2(M_f - 5.01x_1x_2^2)/(435x_2 - 21750) = 0
\] (13)

Where \( j = 3, 5 \) or 6 for moment at mid-span, left support or right support respectively for a doubly reinforced section; \( n = 3, 5 \) or 6 for tension reinforcing bars at the mid-span, left support or right support respectively. Half of the mid-span reinforcement is continuous to the supports which take care of compression reinforcement at the supports.

c) Shear strength requirement

The maximum shear capacity of a beam is given by the following expression (Beckette and Alexandrou, 1997):

\[
V_v = 1/2 Vf_{cd}(0.9bd)
\]

Where \( V_v \) is the maximum shear capacity N/m²

\[
V \text{ is efficiency factor } = 0.7 - f_{y,k}/200 = 0.55
\]

\[
f_{cd} = f_{ck}/1.5 = 20
\]
\[ V \leq V_c = 4.95bd \]

which in terms of design variable implies that
\[ V - 4.125x_1x_2 \leq 0 \]  

(15)

d) Minimum reinforcing steel area constraint,
The EC 2 specifies the minimum reinforcing steel area as:
\[ \rho_{min} = \frac{A_{s,\text{min}}}{b_d d_e} \geq 0.26 \frac{f_{ctm}}{f_{y,c}} \]

Where \( f_{y,c} = 500\text{N/mm}^2 \) and \( f_{ctm} = 2.9\text{N/mm}^2 \)

\[ \frac{0.26 \times 2.9b_b d_d}{500} - A_{s,\text{min}} \leq 0 \]

\[ 0.0015x_1x_2 - A_{s,\text{min}} \leq 0 \]  

(16)

where \( f_{ctm} \) is the mean value of the axial tensile strength of concrete and \( f_{y,c} \) is the design compressive strength of the concrete.

2.2.3 Objective function for column

Let \( A_{g_c} \) be the gross cross sectional area of column; \( x_1 \) and \( x_2 \) be the width \( b_c \) and effective depth \( d_e \) of the beam respectively and \( x_3 \) be the area of longitudinal reinforcement \( A_{s,c} \), then from column details shown in Figure 4, we have the following expressions:

\[ V_{cc} = A_{g_c} L = b_c (d_c + d') (L_c - d_b - d_b') = 140000x_1 + 2800x_1x_2 \]  

(17)

\[ V_{cz} = A_{sc} [L_c - d_b - d_b'] + 1.005 (d_b + d') + 36 d_{ic} = 3871.75x_3 \]  

(18)

Figure 4: Column details

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\[ V_t = A_t \left[ 2(b_t + d_c) - 6d' + 4(d_{sc} + 2d_s) \right] \frac{1}{2} \left\{ L_c - d_b - d'_b \right\} + 1 \]

\[ = 6026x_1 + 6026x_2 - 421820 \]

Therefore,

\[ A_{cf} = 2(b_c + d_c + d')L_u = 2(b_c + d_c + d') \left\{ L_c - d_b - d'_b \right\} = 5600x_1 + 5600x_2 + 280000 \]

And the objective function for the column from Equation (3) is simplified to be:

\[ f = 24000 \times 1.0 \times 9 \times (134585.508 - 5414.492 + 2800 - 3871.75 + 379014.44) + 42 \times 77 \times 1.0 \times 12 \times (5414.492x_1 + 5414.492x_2 + 3871.75x_2 - 379014.44) + 950 \times 1.0 \times 6 (5600x_1 + 5600x_2 + 280000) \]

2.2.4 Column constraint functions

The column constraints based on EC 2 specification are derived in terms of the design variables as follows:

**a) Geometric Constraints**

In order to ensure that the width of the column will not exceed its depth (which is assumed to be in the direction of bonding), the column dimensions are constrained as follows:

\[ b_c \leq h_c \]

\[ b_c - (d_c + d') \leq 0 \]

\[ x_1 - x_2 = 50 \leq 0 \]

(22)

**b) Strength constraint:**

i) **Axial capacity**

The ultimate Load Capacity of a section from EC 2 clause 4.3.5.6.3 is

\[ N_{ud} = 0.567f_{ck}A_c + 0.87A_{sf}f_{yk} \]

The ultimate axial load should be less than the axial capacity of the column. Therefore,

\[ N_{Ed} \leq N_{ud} = 0.567f_{ck}b_c(d_c + d') + 0.87A_{sf}f_{yk} \]

\[ N_{Ed} \geq 850.5x_1 - 17.01x_1x_2 - 435x_3 \]

(23)

where \( N_{Ed} \) and \( N_{ud} \) are ultimate design axial load and axial capacity of the column respectively; \( A_c \) and \( A_{sc} \) are areas of concrete and longitudinal reinforcement respectively. \( b_c, d_c \) and \( A_{sc} \) are represented by \( x_1, x_2 \) and \( x_3 \) respectively.

ii) **Flexural capacity**

A column rectangular reinforced concrete column section is shown in Figure 5 (Mosley, et al. 2007). From the section properties and taking moments about centre of tensile steel,

\[ M_{Ed} = F_{cc}(d - \frac{s}{2}) + F_{sc}(d - d'_c) \]

\[ F_{cc} = 0.567f_{ck}bs \]

\[ F_{sc} = 0.87f_{yk}A'_{s} \]

\[ S = 0.8x = 0.8x0.45d = 0.36d \]

\[ A'_{s} = \frac{d_{sc}}{2} \]

\[ M_{Ed} = 4.184bd^2 + 217.5A_{sc}d - 10875A_{sc} \]

\[ M_{Ed} = 4.184x_1x_2^2 - 217.5x_2x_3 - 10875x_3 \]

(24)

\( F_{cc} \) and \( F_{sc} \) are defined in section 2.2.1. \( M_{cc} \) is the design ultimate moment, \( s \) is the depth of stress block and \( x \) is the depth of neutral axis.

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c) Minimum reinforcing steel area constraint
The minimum reinforcing steel area is provided by the expression:

\[ A_{z,\text{min}} \geq \frac{0.01 N_{Ed}}{0.87 f_{yk}} \geq 0.002 A_c \]

Or

\[ 0.002b(d+d') - A_{z,\text{min}} \leq 0 \]
\[ 0.1x_1 + 0.002x_1x_2 - 452 \leq 0 \]  

(25)

Where \( M_{Ed} \) is the ultimate design moment.

The steps for MATLAB optimization involve creating \( M\)-Files for the objective and constraint functions. Then a constrained optimization routine, \textit{fmincon} that minimizes the objective function is invoked.

3. Results of sensitivity Analysis
The beam spans are varied from 5.4m to 9.4m with 1.0m increment. Service Live Loads were varied from 3.0KN/m\(^2\) to 7.0KN/m\(^2\) with 1.0 KN/m\(^2\) increment. The total of 25 optimal cases was considered. The frame is shown in Figure 6.
3.1 Optimum design variables for beams.
The optimum design variables for beams for span of 5.4m are shown in Table 1.

Table 1: Optimum Beam Design Variables

<table>
<thead>
<tr>
<th>Type of Design</th>
<th>Variable</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
<th>7.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic Design</td>
<td>$X_1=b_b$</td>
<td>225</td>
<td>225</td>
<td>225</td>
<td>225</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>$X_2=d_b$</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>$X_3=A_{sb1}$</td>
<td>539</td>
<td>624</td>
<td>712</td>
<td>790</td>
<td>864</td>
</tr>
<tr>
<td></td>
<td>$X_4=A_{sb2}$</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>$X_5=A_{sb3}$</td>
<td>482</td>
<td>556</td>
<td>632</td>
<td>712</td>
<td>798</td>
</tr>
<tr>
<td></td>
<td>$X_6=A_{sb4}$</td>
<td>1036</td>
<td>1197</td>
<td>1357</td>
<td>1518</td>
<td>1678</td>
</tr>
<tr>
<td></td>
<td>$f=\text{Cost (N)}$</td>
<td>40,776</td>
<td>43,167</td>
<td>45,612</td>
<td>47,926</td>
<td>50,203</td>
</tr>
<tr>
<td>Optimum Design</td>
<td>$X_1=b_b$</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>$X_2=d_b$</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>$X_3=A_{sb1}$</td>
<td>550</td>
<td>640</td>
<td>735</td>
<td>759</td>
<td>833</td>
</tr>
<tr>
<td></td>
<td>$X_4=A_{sb2}$</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td>$X_5=A_{sb3}$</td>
<td>491</td>
<td>568</td>
<td>649</td>
<td>736</td>
<td>753</td>
</tr>
<tr>
<td></td>
<td>$X_6=A_{sb4}$</td>
<td>1065</td>
<td>1226</td>
<td>1386</td>
<td>1487</td>
<td>1675</td>
</tr>
<tr>
<td></td>
<td>$f=\text{Cost (N)}$</td>
<td>38,666</td>
<td>N41,139</td>
<td>N43,716</td>
<td>N44,934</td>
<td>N47,702</td>
</tr>
<tr>
<td>Reduction in cost of beam</td>
<td>5.17%</td>
<td>4.70%</td>
<td>4.16%</td>
<td>6.24%</td>
<td>4.98%</td>
<td></td>
</tr>
</tbody>
</table>

The optimal design results in cost savings of an average of 5.1% as shown in Table 1. It can be seen from Table 1 that the decrease in cost is basically due to decrease in cross section. As the cross section decreased it resulted in increase of areas of reinforcement.

3.2 Optimum cost of beam function
The results for costs of beam are tabulated in Table 2 and graphically represented as shown in Figure 7.

Table 2: Values of Cost of Beam (N)

<table>
<thead>
<tr>
<th>Span of beam (m)</th>
<th>Live Loads (KN/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>5.4</td>
<td>38,666</td>
</tr>
<tr>
<td>6.4</td>
<td>45,579</td>
</tr>
<tr>
<td>7.4</td>
<td>56,997</td>
</tr>
<tr>
<td>8.4</td>
<td>70,275</td>
</tr>
<tr>
<td>9.4</td>
<td>85,445</td>
</tr>
</tbody>
</table>

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The variation in Figure 7 is expressed in the form of a linear equation (Hussanian, 1992)

\[ C_b = C_1 L_b + C_2 \]  

(26)

where \( C_b \) is the cost of beam, \( C_1 \) and \( C_2 \) are coefficients determined from linearly fitting each curve. Linearly fitting the curves of \( C_1 \) gives the coefficients 1866 and 6142. Similarly linearly fitting the curves of \( C_2 \) gives the coefficients -7900 and -3677. Therefore,

\[ C_1 = 1866LL + 6142 \text{ and } C_2 = -7900LL - 3677 \]  

(27)

Where LL is live load in KN/mm²

The cost function is therefore

\[ C_b = C_1 L_b + C_2 = (1866LL+6142) L_b - 7900LL - 3677 \]  

(28)

### 3.3 Optimum span effective depth ratio function

The optimum span effective depth ratios are computed. The results are shown in Table 3 and Figure 8.

<table>
<thead>
<tr>
<th>Span of beam (m)</th>
<th>5.4</th>
<th>6.4</th>
<th>7.4</th>
<th>8.4</th>
<th>9.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span Effective Depth Ratio</td>
<td>18.00</td>
<td>21.33</td>
<td>24.67</td>
<td>28</td>
<td>33.33</td>
</tr>
</tbody>
</table>

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As in the case of cost function, the variation in the curve is expressed in the form of a linear equation, that is:

\[ \frac{L_b}{d_b} = C_1 L_b + C_2 \]

Where \( C_1 \) and \( C_2 \) are determined from linearly fitting the curves. Linearly fitting the curve in Figure 8 gives the Values of \( C_1 \) and \( C_2 \) as 3.733 and -2.558 respectively.

Therefore,

\[ \frac{L_b}{d_b} = C_1 L_b + C_2 = 3.733L_b -2.558 \tag{29} \]

### 3.4 Optimal beam reinforcement ratio function.

The optimal reinforcement ratios for the beam are as shown in Table 4 and plotted as shown in Figure 9.

#### Table 4: Values of Optimum Reinforcement Ratios

<table>
<thead>
<tr>
<th>Span of beam (m)</th>
<th>Live Loads (KN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>5.4</td>
<td>0.009</td>
</tr>
<tr>
<td>6.4</td>
<td>0.013</td>
</tr>
<tr>
<td>7.4</td>
<td>0.019</td>
</tr>
<tr>
<td>8.4</td>
<td>0.025</td>
</tr>
<tr>
<td>9.4</td>
<td>0.032</td>
</tr>
</tbody>
</table>
The optimal beam reinforcement ratio, $\rho$, is also expressed in the form of linear equations as:

$$\rho = C_1 L_b + C_2$$

Again $C_1$ and $C_2$ are determined from linearly fitting each curve. Linearly fitting the curve of values $C_1$ gives the coefficients 0.008 and 0.0035. Similarly linearly fitting values of $C_2$ gives the coefficients 0.0108 and -0.0687. Therefore:

$C_1 = 0.008LL - 0.0035$ and $C_2 = 0.0108LL - 0.0687$.

The reinforcement ratio is therefore

$$\rho = C_1 L_b + C_2 = (0.008LL - 0.0035)L_b + 0.0108LL - 0.0687$$

(31)

### 3.5 Beam Design chart

To plot a beam design chart based on optimum design, $M/\text{bd}^2$, $100A_f/\text{bd}^2$ and $100A_f'/\text{bd}^2$ are computed as presented in Table 5. The optimum design charts are shown in Figures 10 and 11.

**Table 5: Beam Design Chart Data**

<table>
<thead>
<tr>
<th>S/N</th>
<th>Moment, M</th>
<th>$M/\text{bd}^2$</th>
<th>$100A_f/\text{bd}$</th>
<th>$100A_f'/\text{bd}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61.701</td>
<td>3.428</td>
<td>1.067</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>69.835</td>
<td>4.880</td>
<td>1.225</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>77.879</td>
<td>4.327</td>
<td>1.265</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>85.922</td>
<td>4.773</td>
<td>1.330</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>90.175</td>
<td>5.010</td>
<td>1.388</td>
<td>0.230</td>
</tr>
<tr>
<td>6</td>
<td>93.966</td>
<td>5.220</td>
<td>1.508</td>
<td>0.288</td>
</tr>
<tr>
<td>7</td>
<td>102.030</td>
<td>5.668</td>
<td>1.692</td>
<td>0.328</td>
</tr>
<tr>
<td>8</td>
<td>113.885</td>
<td>6.327</td>
<td>1.857</td>
<td>0.532</td>
</tr>
<tr>
<td>9</td>
<td>124.532</td>
<td>6.918</td>
<td>1.873</td>
<td>0.737</td>
</tr>
<tr>
<td>10</td>
<td>125.740</td>
<td>6.986</td>
<td>2.057</td>
<td>0.757</td>
</tr>
</tbody>
</table>
Figure 10: Typical optimum beam design chart for tension reinforcement
3.6 Column Design Variables
The deterministic as well as the optimum column design results are as presented in Table 6.

Table 6: Deterministic and Optimum Column Design Results

<table>
<thead>
<tr>
<th>Type of Design</th>
<th>Variable</th>
<th>Live load (KN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>Deterministic</td>
<td>X₁=b₁</td>
<td>225</td>
</tr>
<tr>
<td>Design</td>
<td>X₂=d₁</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>X₃=A₁c</td>
<td>452</td>
</tr>
<tr>
<td></td>
<td>f=Cost (N)</td>
<td>15,118</td>
</tr>
<tr>
<td>Optimum Design</td>
<td>X₁=b₁</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>X₂=d₁</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>X₃=A₁c</td>
<td>452</td>
</tr>
<tr>
<td></td>
<td>f=Cost (N)</td>
<td>14,730</td>
</tr>
<tr>
<td>Reduction in cost</td>
<td>2.57%</td>
<td>2.57%</td>
</tr>
</tbody>
</table>

4. Discussion
The principles involved in the direct stiffness method and the procedure for optimum design of RC frames are presented in this study. It is seen the design variables pertaining to concrete dimensions are at their upper bounds. The active constraints on the beam and column are those imposed on the axial capacity, flexural capacity and minimum steel area. The optimal design results in cost savings of an average of 5.1% for the beam and 3.2% for the column. The decrease in cost is basically due to decrease in cross section. As the cross section decreased it resulted in increase of areas of reinforcement.

Figure 11: Typical optimum beam design chart for compression reinforcement
The decrease in cost is lower in the column. This can be attributed to the fewer number of design variables in column optimization. Expressions for computations of span effective depth and reinforcement ratios are obtained. Furthermore beam design charts are obtained for optimum design of reinforced concrete beams. The expressions and design charts based on optimum design concept will be valuable for preliminary and low cost design of reinforced concrete structures.

5. Conclusion
The stiffness method is used to analyze a two bay one storey plane frame. A procedure for optimization design of the frame is presented. The optimization procedure minimizes the cost of reinforced concrete while satisfying the limitation and specification of Eurocode 2. The optimal design results in cost savings of an average of 5.1% for the beam and 3.2% for the column. The saving in cost is lower in the column. This can be attributed to the fewer number of variables in column design. Expressions for computations of span effective depth and reinforcement ratios are obtained. Furthermore beam design charts are obtained for optimum design of reinforced concrete beams. The expressions and design charts based on optimum design concept will be valuable for preliminary and low cost design of reinforced concrete structures.

REFERENCES