# Absolute Difference of Square Sum and Sum Mean Prime Labeling of Tree Graphs

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## ABSTRACT

Absolute difference of square sum and sum mean prime labeling of a graph is the labeling of the vertices with {1, 2---, p} and the edges with absolute difference of the mean of the squares of the labels of the incident vertices and the sum of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits absolute difference of square sum and sum mean prime labeling. Here we investigate some tree graphs for absolute difference of square sum and sum mean prime labeling,

***Keywords*** - Graph labeling, square sum, prime labeling, prime graphs, trees.

## I. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In this paper we investigated absolute difference of square sum and sum mean prime labeling of some tree graphs.

**Definition: 1.1** Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

## II. MAIN RESULTS

**Definition 2.1** Let G = (V (G), E (G)) be a graph with p vertices and q edges. Define a bijection
f: V(G) → {1,2,---, p} by $f(v_i) = i$ , for every i from 1 to p and define a 1-1 mapping
$f^*_{adsssmp}$ : E(G) → set of natural numbers N by
$f^*_{adsssmp}(uv) = \frac{1}{2}\,|\, f(u)^2 + f(v)^2 - \{f(u)+f(v)\}|$.
The induced function $f^*_{adsssmp}$ is said to be an absolute difference of square sum and sum mean prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

**Definition 2.2** A graph which admits absolute difference of square sum and sum mean prime labeling is called an absolute difference of square sum and sum mean prime graph.

**Theorem 2.1** Comb graph $P_n \odot K_1 (n > 2)$ admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let G = $P_n$ and let $v_1,v_2,\text{---},v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n-1.
Define a function f : V → {1,2,---,2n} by
$f(v_i) = i$ , i = 1,2,---,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows
$f^*_{adsssmp}(v_i\, v_{i+1}) = i^2$, i = 1,2,---,n+1.
$f^*_{adsssmp}(v_{i+2}\, v_{2n+1-i}) = \frac{(2n+1-i)^2+(i+2)^2-2n-3}{2}$ ,
i = 1,2,---,n-2.
Clearly $f^*_{adsssmp}$ is an injection.

$$
\begin{aligned}
\textbf{\textit{gcin}} \text{ of } (v_{i+1}) &= \text{gcd of } \{f^*_{adsssmp}(v_i\, v_{i+1}), f^*_{adsssmp}(v_{i+1}\, v_{i+2})\} \\
&= \text{gcd of } \{i^2, (i+1)^2\} \\
&= \text{gcd of } \{i, i+1\} \\
&= 1, \qquad i = 1,2,\text{---},n.
\end{aligned}
$$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \Theta K_1$ , admits absolute difference of square sum and sum mean prime labeling.

**Example 2.1** $G = P_4 \Theta K_1$

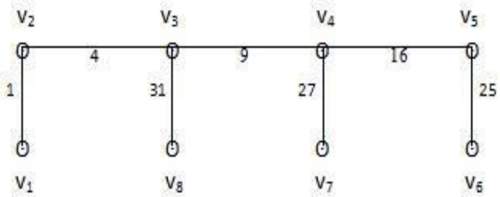


fig 2.1

**Theorem 2.2** Centipede graph $P_n \Theta 2K_1 (n > 2)$ admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let $G = P_n$ and let $v_1, v_2, ---, v_{3n}$ are the vertices of G.

Here $|V(G)| = 3n$ and $|E(G)| = 3n-1$.

Define a function $f : V \to \{1,2,---,3n\}$ by

$$f(v_i) = i, \; i = 1,2,---,3n$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows

$f^*_{adsssmp}(v_{3i-2}\, v_{3i-1}) = (3i-2)^2$, $i = 1,2,---,n$.

$f^*_{adsssmp}(v_{3i-1}\, v_{3i}) = (3i-1)^2$, $i = 1,2,---,n$.

$f^*_{adsssmp}(v_{3i-1}\, v_{3i+2}) = 9i^2+2$, $i = 1,2,---,n-1$.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of $(v_{3i-1})$ = gcd of $\{f^*_{adsssmp}(v_{3i-2}\, v_{3i-1}), f^*_{adsssmp}(v_{3i-1}\, v_{3i})\}$
= gcd of $\{(3i-2)^2, (3i-1)^2\}$
= gcd of $\{3i-2, 3i-1\}$
$=1$, $i = 1,2,---,n$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \Theta 2K_1$, admits absolute difference of square sum and sum mean prime labeling.

**Example 2.2** $G = P_n \Theta 2K_1$

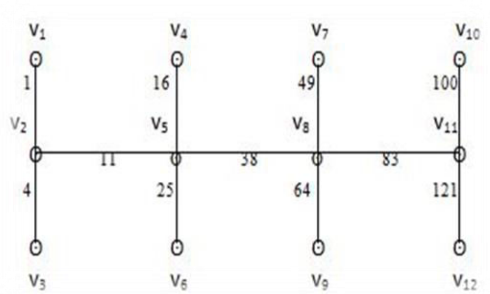


fig – 2.2

**Theorem 2.3** Twig graph $(n > 2)$ admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let $G = T_W(n)$ and let $v_1, v_2, ---, v_{3n-4}$ are the vertices of G.

Here $|V(G)| = 3n-4$ and $|E(G)| = 3n-5$.

Define a function $f : V \to \{1,2,---,3n-4\}$ by

$$f(v_i) = i, \; i = 1,2,---,3n-4.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows

$f^*_{adsssmp}(v_i\, v_{i+1}) = i^2$, $i = 1,2,---,n-1$.

$f^*_{adsssmp}(v_{n-i}\, v_{2n-2+i}) = \frac{(2n-2+i)^2+(n-i)^2-3n+2}{2}$, $i = 1,2,---,n-2$.

$f^*_{adsssmp}(v_{n-i}\, v_{n+i}) = \frac{(n+i)^2+(n-i)^2-2n}{2}$, $i = 1,2,---,n-2$.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of $(v_{i+1})$ = gcd of $\{f^*_{adsssmp}(v_i\, v_{i+1}), f^*_{adsssmp}(v_{i+1}\, v_{i+2})\}$
= gcd of $\{i^2, (i+1)^2\}$
= gcd of $\{i, i+1\}$
$=1$, $i = 1,2,---,n-2$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $T_W(n)$, admits absolute difference of square sum and sum mean prime labeling.

**Example 2.3** $G = T_W(4)$

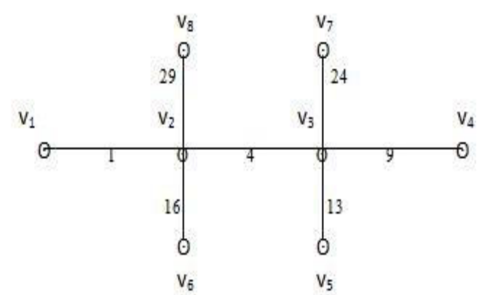


fig -2.3

**Theorem 2.4** $P_n \Theta 3K_1 (n > 2)$ admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let $G = P_n \Theta 3K_1$ and let $v_1, v_2, ---, v_{4n}$ are the vertices of G.

Here $|V(G)| = 4n$ and $|E(G)| = 4n-1$.

Define a function $f : V \to \{1,2,---,4n\}$ by

$$f(v_i) = i, \; i = 1,2,---,4n$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows

$f^*_{adsssmp}(v_{4i-3}\, v_{4i-2}) = (4i-3)^2$, $i = 1,2,---,n$.

$f^*_{adsssmp}(v_{4i-2}\ v_{4i-1})$ = $(4i-2)^2$,
i = 1,2,---,n.
$f^*_{adsssmp}(v_{4i-2}\ v_{4i})$ = $16i^2-12i+3$,
i = 1,2,---,n.
$f^*_{adsssmp}(v_{4i-2}\ v_{4i+2})$ = $16i^2-4i+4$,
i = 1,2,---,n-1.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of $(v_{4i-2})$ = gcd of $\{f^*_{adsssmp}(v_{4i-3}\ v_{4i-2}), f^*_{adsssmp}(v_{4i-2}\ v_{4i-1})\}$
= gcd of $\{(4i-3)^2, (4i-2)^2\}$
= gcd of {4i-3,4i-2}
=1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $P_n \odot 3K_1$, admits absolute difference of square sum and sum mean prime labeling.

**Theorem 2.5** H- graph of path $P_n$ (n > 2) admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let G = $H(P_n)$ and let $v_1,v_2,\text{---},v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n-1.
Define a function f : V → {1,2,---,2n} by
$f(v_i)$ = i , i = 1,2,---,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows
$f^*_{adsssmp}(v_i\ v_{i+1})$ = $i^2$, i = 1,2,---,n-1.
$f^*_{adsssmp}(v_{n+i}\ v_{n+i+1})$ = $(n+i+1)^2$,
i = 1,2,---,n-1.
$f^*_{adsssmp}(v_{\frac{n+1}{2}}\ v_{\frac{3n+1}{2}})$ = $\frac{5n^2-1}{4}$, n is odd.
$f^*_{adsssmp}(v_{\frac{n+2}{2}}\ v_{\frac{3n}{2}})$ = $\frac{5n^2-2n}{4}$, n is even.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of $(v_{i+1})$ = gcd of $\{f^*_{adsssmp}(v_i\ v_{i+1}), f^*_{adsssmp}(v_{i+1}\ v_{i+2})\}$
= gcd of $\{i^2, (i+1)^2\}$
= gcd of {i,i+1}=1,
i = 1,2,---,n-2.

***gcin*** of $(v_{n+i+1})$ = gcd of $\{f^*_{adsssmp}(v_{n+i}\ v_{n+i+1}), f^*_{adsssmp}(v_{n+i+1}\ v_{n+i+2})\}$
= gcd of $\{(n+i)^2, (n+i+1)^2\}$
= gcd of {n+i,n+i+1}
=1, i = 1,2,---,n-2.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $H(P_n)$, admits absolute difference of square sum and sum mean prime labeling.

**Theorem 2.6** $H(P_n) \odot K_1$ (n > 2) admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let G = $H(P_n)$ and let $v_1,v_2,\text{---},v_{4n}$ are the vertices of G.
Here |V(G)| = 4n and |E(G)| = 4n-1.
Define a function f : V → {1,2,---,4n} by
$f(v_i)$ = i , i = 1,2,---,4n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows
$f^*_{adsssmp}(v_i\ v_{i+1})$ = $i^2$, i = 1,2,---,n+1.
$f^*_{adsssmp}(v_{i+2}\ v_{2n+1-i}) = \frac{(2n+1-i)^2+(i+2)^2-2n-3}{2}$,
i = 1,2,---,n-2.

$f^*_{adsssmp}(v_{2n+i}\ v_{2n+i+1})$ = $(2n+i)^2$,
i = 1,2,---,n+1.
$f^*_{adsssmp}(v_{2n+i+2}\ v_{4n+1-i}) = \frac{(4n+1-i)^2+(2n+i+2)^2-6n-3}{2}$,
i = 1,2,---,n-2.
$f^*_{adsssmp}(v_{\frac{n+3}{2}}\ v_{\frac{5n+3}{2}}) = \frac{\frac{(n+3)^2}{4}+\frac{(5n+3)^2}{4}-3n-3}{2}$,
n is odd.
$f^*_{adsssmp}(v_{\frac{n+4}{2}}\ v_{\frac{5n+2}{2}}) = \frac{\frac{(n+4)^2}{4}+\frac{(5n+2)^2}{4}-3n-3}{2}$,
n is even.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of $(v_{i+1})$ = gcd of $\{f^*_{adsssmp}(v_i\ v_{i+1}), f^*_{adsssmp}(v_{i+1}\ v_{i+2})\}$
= gcd of $\{i^2, (i+1)^2\}$
= gcd of {i,i+1}
=1, i = 1,2,---,n.

***gcin*** of $(v_{2n+i+1})$ = gcd of $\{f^*_{adsssmp}(v_{2n+i}\ v_{2n+i+1}), f^*_{adsssmp}(v_{2n+i+1}\ v_{2n+i+2})\}$
= gcd of $\{(2n+i)^2, (2n+i+1)^2\}$
= gcd of {2n+i,2n+i+1}
=1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $H(P_n) \odot K_1$, admits absolute difference of square sum and sum mean prime labeling.

**Theorem 2.7** Coconut tree graph CT(m,n) (m, n >2) admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let G = CT(m,n) and let $v_1,v_2,\text{---},v_{m+n}$ are the vertices of G.
Here |V(G)| = m+n and |E(G)| = m+n-1.
Define a function f : V → {1,2,---,m+n} by
$f(v_i)$ = i , i = 1,2,---,m+n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows
$f^*_{adsssmp}(v_i\ v_{i+1})$ = $i^2$, i = 1,2,---,m.

$f^*_{adsssmp}(v_m\, v_{m+i}) = \frac{(m+i)^2+(m)^2-2m-i}{2}$,
i = 1,2,---,n-1.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of ($v_{i+1}$) = gcd of $\{f^*_{adsssmp}(v_i\, v_{i+1}), f^*_{adsssmp}(v_{i+1}\, v_{i+2})\}$
= gcd of $\{i^2, (i+1)^2\}$
= gcd of {i,i+1}
=1, i = 1,2,---,m-1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence CT(m,n), admits absolute difference of square sum and sum mean prime labeling.

**Theorem 2.8** Double coconut tree graph DCT(m,n,k) (m,n.k > 2) admits absolute difference of square sum and sum mean prime labeling.

**Proof:** Let G = DCT(m,n,k) and let $v_1, v_2$,---,$v_{m+n+k}$ are the vertices of G.
Here |V(G)| = m+n+k and |E(G)| = m+n+k-1.
Define a function f : V → {1,2,---,m+n+k} by
$f(v_i)$ = i , i = 1,2,---,m+n+k.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{adsssmp}$ is defined as follows

$f^*_{adsssmp}(v_{m+1}\, v_i) = \frac{(m+1)^2+(i)^2-m-i-1}{2}$,
i = 1,2,---,m.

$f^*_{adsssmp}(v_{m+i}\, v_{m+i+1}) = (m+i)^2$,
i = 1,2,---,n-1.

$f^*_{adsssmp}(v_{m+n}\, v_{m+n+i}) = \frac{(m+n+i)^2+(m+n)^2-2m-2n-i}{2}$,
i = 1,2,---,k.

Clearly $f^*_{adsssmp}$ is an injection.

***gcin*** of ($v_{m+i+1}$) = gcd of $\{f^*_{adsssmp}(v_{m+i}\, v_{m+i+1}), f^*_{adsssmp}(v_{m+i+1}\, v_{m+i+2})\}$
= gcd of $\{(m+i)^2, (m+i+1)^2\}$
= gcd of {m+i,m+i+1}
=1, i = 1,2,---,n.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence DCT(m,n,k), admits absolute difference of square sum and sum mean prime labeling.

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