POWER OUTPUT MODELLING: A PANACEA FOR NIGERIA'S ENERGY SUSTAINABILITY

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ABSTRACT
Recent efforts made by the successive Nigerian governments to ensure constant and reliable electricity supply in the country has been hampered by many factors among which are inappropriate planning and unpredictability of the power resource. It is on this premise that this paper seeks to analyze the power output from Shiroro hydropower station, using a merger of two categories of daily readings into one entity, in order to obtain an average value for daily variables. Ten years hydrological data from Shiroro power station representing 3653 daily inputs across seven independent variables namely: Head water elevation, Tail water elevation, Gross operating head, Storage volume, Storage differential, Average discharge, and Computed inflow, all denoted by x_{i-n}; where i = 1 to 7, were obtained and analyzed using Multivariate Regression Analysis (MRA). The result generated showed appropriate model which can be used to predict power output from the station and also ensure adequate planning for government and non-governmental agencies. However, with a meritorious R^2 value of 0.97, the MAPE and MPE of 79.18% raised great concern as regards to the model's efficacy. This further opens up new areas of research interest for scholars and organizational heads alike, in terms of efficient energy management.

***Keywords**: Gross operating head, efficiency, head water elevation, multivariate linear regression, average turbine discharge, power output*

I. INTRODUCTION
The high demand for electricity for industrial and domestic use has necessitated the need for stable and continuous power supply source to the consumers. Therefore, any measure taken to improve operational performance of a nation's electric supply is apt, as this translates to economic growth and development of such a country or nation. The major hydropower stations within Nigeria have failed to meet the ever increasing demand for electricity by consumers due to unplanned management schemes. It is for this reason that several efforts have been made over the years to improve power supply performance of Power Holding Company of Nigeria (PHCN).
Hydropower, besides being emission free and renewable, has the most operating benefits that provide enhanced value to the electric system in the form of efficiency, security, maintenance free and most importantly, reliability. The abundance of water resource and its renewability makes this a national asset. The way and manner in which hydraulic forces vary to affect power output is not well understood. Besides, the glaring perturbations in the hydraulic variates due to seasonal changes and its attendant effects, equally raises great concern to energy planners on variations in power output as a result of this phenomena.
Generating stations are an integral part of the entire power system chain in the country. Their optimal performance and reliability is key to the sustainability of the power industry. Furthermore, the reliability of these stations is a function of the generating units within the station. Adequate planning of the generating capacity of these generating units is a prelude to system improvement for futuristic operations. This prediction is aimed at meeting the growing consumer demand.

II. LITERATURE REVIEW

Ramani and Rom (2007), adopted different methods in the control of unpredicted and non-deterministicnature of hydraulic parameters.However, according to Kishor, Saini, and Singh (2007), adequate regulation of turbine poses greater challenge in control system in hydropower station. They further recommended that there is need for an advanced modeling and control technique for more effective control. A simplified algorithm was developed by Abbas, Saleem, and Ali (2011), using Fuzzy logic approach which classified water level, flow rate, release valve and drain valve control, as input and output parameters. Masden*et al* (2009), opined that hydrological and hydraulic simulation models were coupled with numerical optimization algorithm as an effective tool for reservoir optimization by different management objectives.
Vieira and Ramos (2009), applied linear programming and MATLAB in a water system of a pumping station, which influenced the choice of this approach to forecast for a dam or power station. Again, this approach adopted all hydrological information within the dam, as independent variables responsible for power output. It further dealt with two varied readings/recordings of morning and night records by merging them together. The average of both readings was chosen to ensure a better representation of all samples within population. Also, data span of ten years which ensured variability and changes in trend were equally captured. This approach will develop an operating model base to enhance productivity, planning and forecasting efforts, to meet consumer and industrial needs.
Charles, Kelly, and Richard (2015),considered anthropogenic and climate conditions in reservoir construction while Brett *et al* (2004), tried to forecast reservoir thermal conditions by employing one dimensional thermal model to stimulate the thermal effect of a new dam. Petras*et al* (2011), applied geographic information system (GIS) software to predict energy output of a hydro-station using head and flow duration data that gives the time variability of water discharge. A gamut of variables in hydro dam operation were compiled by Ifabiyi (2011a) using multiple regression, factor stepwise regression method and factor analysis in the Jebba hydro-station.Abdulkadir*et al* (2013),admitted that efficient management of hydropower reservoir variables is apt and used multilayer perceptron neural network to model other key variables across two hydropower dams.
The emphasis made by Suleiman and Ifabiyi (2014), that efficient reservoir operation and management for operational decision making by arrayingof hydro-meteorological elements within the dam area using statistical analysis of correlation and regression analysis to determine the spatio-temporal trend. While the need for efficient interplay between reservoir elements for sustainable power development of hydro dam capacities was stressed by Ifabiyi (2011b), using descriptive and inferential statistics within the array of identified elements, by PCA(Principal Component Analysis).
It is therefore clear that none of these literature reviews have clearly sort to establish a clear interplay of variables' roles in relation to power output by considering the duality of daily records in Morning and Night readings within the Shiroro dam. This in essence is to give a better representation of distinctive information within that sample population size, instead of individual entities alone.

III. METHODS AND MATERIALS

The research design adopted a merger of two categories of daily readings into one entity, in order to obtain an average value for daily variables. A total of ten years daily hydrological data were used in this analysis, representing 3653 daily inputs across seven independent variables namely; Head water elevation, Tail water elevation, gross operating head, storage volume, storage differential, Average discharge, and computed inflow, all denoted by x_{i-n}; where i = 1 to 7. And the dependent variable, power output, tagged as y.

The multivariate linear Regression model is given as;
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \varepsilon_i \quad (1)$$
It is instructive to note that the values of $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ and y have been earlier determined as stated above. This follows therefore that;
$$\Sigma y = \Sigma\beta_0 + \Sigma\beta_1 x_1 + \Sigma\beta_2 x_2 + \Sigma\beta_3 x_3 + \Sigma\beta_4 x_4 + \Sigma\beta_5 x_5 + \Sigma\beta_6 x_6 + \Sigma\beta_7 x_7 + \Sigma\varepsilon_i \quad (2)$$
Whereas; $\Sigma\beta_0 = n\beta_0$ and $\Sigma\varepsilon_i = 0$
Further decomposition of equation (2) will be shown as
$$\Sigma x_1 y = \beta_0\Sigma x_1 + \beta_1\Sigma x_1^2 + \beta_2\Sigma x_1\Sigma x_2 + \beta_3\Sigma x_1\Sigma x_3 + \beta_4\Sigma x_1\Sigma x_4 + \beta_5\Sigma x_1\Sigma x_5 + \beta_6\Sigma x_1\Sigma x_6 + \beta_7\Sigma x_1\Sigma x_7 \quad (3)$$
$$\Sigma x_2 y = \beta_0\Sigma x_2 + \beta_1\Sigma x_2\Sigma x_1 + \beta_2\Sigma x_2^2 + \beta_3\Sigma x_2\Sigma x_3 + \beta_4\Sigma x_2\Sigma x_4 + \beta_5\Sigma x_2\Sigma x_5 + \beta_6\Sigma x_2\Sigma x_6 + \beta_7\Sigma x_2\Sigma x_7 \quad (4)$$
$$\Sigma x_3 y = \beta_0\Sigma x_3 + \beta_1\Sigma x_3\Sigma x_1 + \beta_2\Sigma x_3\Sigma x_2 + \beta_3\Sigma x_3^2 + \beta_4\Sigma x_3\Sigma x_4 + \beta_5\Sigma x_3\Sigma x_5 + \beta_6\Sigma x_3\Sigma x_6 + \beta_7\Sigma x_3\Sigma x_7 \quad (5)$$
$$\Sigma x_4 y = \beta_0\Sigma x_4 + \beta_1\Sigma x_4\Sigma x_1 + \beta_2\Sigma x_4\Sigma x_2 + \beta_3\Sigma x_4\Sigma x_3 + \beta_4\Sigma x_4^2 + \beta_5\Sigma x_4\Sigma x_5 + \beta_6\Sigma x_4\Sigma x_6 + \beta_7\Sigma x_4\Sigma x_7 \quad (6)$$

$\Sigma x_5 y = \beta_0\Sigma x_5 + \beta_1\Sigma x_5\Sigma x_1 + \beta_2\Sigma x_5\Sigma x_2 + \beta_3\Sigma x_5\Sigma x_3 + \beta_4\Sigma x_5\Sigma x_4 + \beta_5\Sigma x_5^2 + \beta_6\Sigma x_5\Sigma x_6 + \beta_7\Sigma x_5\Sigma x_7$ (7)
$\Sigma x_6 y = \beta_0\Sigma x_6 + \beta_1\Sigma x_6\Sigma x_1 + \beta_2\Sigma x_6\Sigma x_2 + \beta_3\Sigma x_6\Sigma x_3 + \beta_4\Sigma x_6\Sigma x_4 + \beta_5\Sigma x_6\Sigma x_5 + \beta_6\Sigma x_6^2 + \beta_7\Sigma x_6\Sigma x_7$ (8)
$\Sigma x_7 y = \beta_0\Sigma x_7 + \beta_1\Sigma x_7\Sigma x_1 + \beta_2\Sigma x_7\Sigma x_2 + \beta_3\Sigma x_7\Sigma x_3 + \beta_4\Sigma x_7\Sigma x_4 + \beta_5\Sigma x_7\Sigma x_5 + \beta_6\Sigma x_7\Sigma x_6 + \beta_7\Sigma x_7^2$(9)

The above summations were computed and developed into a matrix form as shown below for solution determination using Matlab.

$$\begin{bmatrix} a_{11}\ a_{12}\ a_{13}\ a_{14} \cdots a_{18} \ !\ b_1 \\ a_{21}\ a_{22}\ a_{23}\ a_{24} \cdots a_{28} \ !\ b_2 \\ a_{31}\ a_{32}\ a_{33}\ a_{34} \cdots a_{38} \ !\ b_3 \\ a_{41}\ a_{42}\ a_{43}\ a_{44} \cdots a_{48} \ !\ b_4 \\ a_{51}\ a_{52}\ a_{53}\ a_{54} \cdots a_{58} \ !\ b_5 \\ a_{61}\ a_{62}\ a_{63}\ a_{64} \cdots a_{68} \ !\ b_6 \\ a_{71}\ a_{72}\ a_{73}\ a_{74} \cdots a_{78} \ !\ b_7 \\ a_{81}\ a_{82}\ a_{83}\ a_{84} \cdots a_{88} \ !\ b_8 \end{bmatrix}$$

IV. RESULT AND DISCUSSION

The result obtained from the matrix using Matlab software, gave rise to table 1,as solution to 3653 average daily hydrological data recordings in the model below.

Table 1: Obtained values for the matrix

a_{1i}	a_{2i}	a_{3i}	a_{4i}	a_{5i}	a_{6i}	a_{7i}	a_{8i}	$b_{1\text{--------}8}$
3653	1348057.8	987614.15	360445.87	11166.34	0.59	1100232.5	1123678	23197276
1348057	497737368.77	364467854.64	133270337.86	4167301.35	199.97	407856326.98	417440628.63	8616328208.84
987614.15	364467854.64	267010187.97	97458266.37	3020775.89	149.81	297726237.05	3043259000.48	6277946248.45
360445.87	133270337.86	97458266.37	35812297.93	1146533.07	50.21	1101306587.77	113114804.47	2338394315.18
11166.34	4167301.35	3020775.89	1146533.07	42568.2	-2.55	3670550.78	3921489.55	80539913.52
0.59	199.97	149.81	50.2	-2.55	11.83	-479.1	-4724.46	-74290.73
1100232.5	407856326.98	297726237.05	1101306587.77	3670550.78	-479.1	402043943.25	459020944.5	8396031017
11236.78	417440628.63	3043259000.48	113114804.47	3921489.55	-4724.46	459020944.5	1121861854	9534437293

Eigen Value Determination Using Matlab

A = 1.0e+011 *

```
0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
0.0000  0.0050  0.0036  0.0013  0.0000  0.0000  0.0041  0.0042
```

```
    0.0000    3.6447    0.0027    0.0010    0.0000    0.0000    0.0030    0.0030
    0.0000    0.0013    0.0010    0.0004    0.0000    0.0000    0.0110    0.0011
    0.0000    0.0000    0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000   -0.0000
    0.0000    0.0041    0.0030    0.0110    0.0000   -0.0000    0.0040    0.0046
    0.0000    0.0042    0.0304    0.0011    0.0000   -0.0000    0.0046    0.0112

>> B = inv(A)


B =

   -0.0000    0.0000   -0.0000   -0.0000   -0.0004   -0.0006   -0.0000   -0.0000
   -0.0000   -0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
   -0.0000    0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000    0.0000
   -0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000
   -0.0001    0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.0006   -0.0000   -0.0000   -0.0000    0.0007    0.0848    0.0000   -0.0000
    0.0000   -0.0000   -0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.0000   -0.0000   -0.0000   -0.0000    0.0000    0.0000    0.0000   -0.0000

>> C = [23197276; 8616328208.84; 6277946248.45; 2338394315.18; 80539913.52; -74290.73; 8396031017; 9534437293]

C =  1.0e+009 *

    0.0232
    8.6163
    6.2779
    2.3384
    0.0805
   -0.0001
    8.3960
    9.5344

>> D = B*C

D =  1.0e+003 *

    2.0041
   -0.0000
   -0.0042
   -0.0008
    0.0016
    0.9042
    0.0000
    0.0181
```

$$\hat{y} = 2004.1 - 0x_1 - 4.2x_2 - 0.8x_3 - 1.6x_4 + 904.2x_5 + 0x_6 + 18.1x_7 \quad (8)$$

Interpretation of model coefficients

The analysis and modelling returned x_1 and $x_7$as zero. They represent Head water elevation and average turbine discharge, respectively.

1. Head water elevation ($x_1 = 0$) implies that it has no direct significance to the operation process (i.e energy production) whether it is high or low. Whereas the key determinant for operation is the gross operating water head which is a key parameter necessary for production.

2. Average Turbine Discharge (x_6) variable is zero because this is water that leaves the reservoir after work done of impinging on the turbine impellers to produce energy (power). It could also be termed a "waste product" of the entire production process and therefore returned zero.

3. Storage Differential (x_5) variable attracts a high coefficient of nearly 5.0 in 4.7. This implies that the daily changes in volume (especially negative changes as in reduction in volume), contribute immensely to power production. This means that every $1m^3$ drop in storage volume, translates to nearly five times the amount of power that would have been produced if there was no such loss in volume. There is therefore an urgent need to address or augment for these losses to maintain constant daily volume of water for production.

4. Computed inflow (x_7) is also negative but insignificant with 0.0121. This shows that as long as the water level is above the dead storage level and gross operating head still exists, that production must continue at all times. However, the negative further highlights the unhealthy and significant nature of the variable as it is a case of production inventory stock depletion, without restocking. This will result in a shutdown, when stock is empty.

Model Fitting and Diagonistic

The model developed was fitted to 31 daily observations as specimen for model testing. The resulting forecast is depicted in table 2.

Table 2: Forecast results using 31 days as specimen

y	ŷ	e_t	$/e_t/$	e_t^2	$P\varepsilon_t$	σ	σ^2
11059	768	10291	10291	105903619	93.05%	2836.87	8047837
8307	1544	6763	6763	45734964	81.41%	84.87	7203
7187	1423	5764	5764	33218833	80.19%	-1035.13	1071494
7040	1567	5473	5473	29950860	77.74%	-1182.13	1397431
7187	1271	5916	5916	35003453	82.32%	-1035.13	1071494
7374	1433	5941	5941	35293285	80.56%	-848.13	719324
8304	3734	4570	4570	20887070	55.04%	81.87	6703
6907	623	6284	6284	39484963	90.98%	-1315.13	1729567
7788	2445	5343	5343	28552833	68.61%	-434.13	188469
7770	1868	5902	5902	34834567	75.96%	-452.13	204422
7443	2660	4783	4783	22877168	64.26%	-779.13	607044
7579	869	6710	6710	45018628	88.53%	-643.13	413616
7090	470	6620	6620	43829571	93.38%	-1132.13	1281718
8060	1275	6785	6785	46036473	84.18%	-162.13	26286
7600	2345	5255	5255	27611293	69.14%	-622.13	387046
8171	1446	6725	6725	45219770	82.30%	-51.13	2614
9122	360	8762	8762	76772733	96.05%	899.87	809766
7847	3040	4807	4807	23107590	61.26%	-375.13	140723
7338	679	6659	6659	44338312	90.74%	-884.13	781686
7601	2049	5552	5552	30823024	73.04%	-621.13	385802
7498	1860	5638	5638	31788512	75.20%	-724.13	524364

8095	2041	6054	6054	36645070	74.78%	-127.13	16162
8161	1366	6795	6795	46169879	83.26%	-61.13	3737
8649	3008	5641	5641	31822239	65.22%	426.87	182218
9554	2731	6823	6823	46551682	71.41%	1331.87	1773878
7713	899	6814	6814	46435351	88.35%	-509.13	259213
8868	2235	6633	6633	43991338	74.79%	645.87	417148
8937	2677	6260	6260	39192014	70.05%	714.87	511039
10877	1856	9021	9021	81369759	82.93%	2654.87	7048335
10763	1821	8942	8942	79960229	83.08%	2540.87	6456020
8997	291	8706	8706	75788351	96.76%	774.87	600424
				MPE	**79.18%**		
				MAPE	**79.18%**		

Test for model adequacy

Arising from the outcome of table 2, it has become imperative to test for adequacy of model as a predictive tool.
The coefficient of determination (R^2) is given by

$$R^2 = 1 - \frac{\left[\sum \left(y_i - \hat{y}\right)^2\right]}{\left[\sum \left(y_i - \bar{y}\right)^2\right]} = 0.97$$

Furthermore the coefficient of correlation (R) is given as

$$R = \sqrt{R^2} = 0.99$$

V. CONCLUSION AND RECOMMENDATION

The model developed was successful in predicting power output from the Shiroro dam using ten year hydrological data. The results obtained further showed the role of individual variable contribution within the production system to the power output. The degree of association between these variables was also found to be meritorious. Thus by adopting this model, a near accurate forecast of future demands is made. In addition to this foregoing development, various attempts at power planning could be achieved to meet consumer and industrial energy requirement which isa key ingredient to industrial revolution efforts within the country and the world at large.

This approach has further clarified the way and manner hydraulic forces variations affect hydropower output. This will enhance power improvement and open up new concept for efficient energy planning for national growth, development and sustainability. On the other hand, areas of research interests in the application of time series forecasting model and analysis have been broadened by this expository work. This is aimed at addressing the observed flaws and further reduces the error term, while also accounting for the dynamic nature of a production system such as this.

REFERENCES

[1] Ramani, V. and Rom, M. (2007), "Fuzzy Logic Model on Operation and control of Hydropower dams in Malaysia" ICCES VOL. 4

[2] Kishor, N., Saini, R., and Singh S.(2007),"A review on Hydropower plant models and control" Science Direct Renewable and SustainableEnergyReviews 11 (2007) 776–796 www.elsevier.com/locate/rser

[3] Abbas, M., Saleem, K., and Ali, N. (2011), "Fuzzy Logic Based Hydro-electric Power Dam Control System" International Journal of Scientific & Engineering Research Volume 2, Issue 6

[4] Masden, H., Richaud, B., Pedersen, C., and Borden, D.(2009), "A real-time flow Forecasting and Reservoir Optimization System for optimizing Hydropower production" Waterpower XVI

[5] Vieira, F., and Ramos, H.(2009), "Optimization of operational planning for wind/hydro hybrid water supply systems" www.elsevier.com/locate/renene Renewable Energy 34

[6] Charles, N., Kelly, E., and Richard, M.(2015), "Hypothesis for hydrologic alteration" Elsevier journal of Hydrology Vol. 5, iss. 30

[7] Brett, M., Laurel. B., Anderson, A., Paul, W., Mary A., and Darell, F.(2004), "Effects of Climate and Dam Operations on Reservoir Thermal Structure" Journal of water Resources Planning and Management, Vol. 130, No. 2

[8] Petras P., Antanas, D., Algis, K., and Gitana, K.(2011), "Tools for Small Hydropower Plant Resource Planning and Development: A Review of Technology and Applications". www.mdpi.com/journal/energies. Energies 2011,Vol 4

[9] Ifabiyi, I. (2011a) "Relationship between Power generation and reservoir Elements in the Jebba hydroelectric Reservoir, Nigeria" Global Journal of science Frontier, Volume 11 issue 8

[10] Abdulkadir T., Salami A., Anwar, A., and Kareem A.(2013), "Modelling of Hydropower Reservoir variables for energy Generation: Neural Network approach" Ethiopian journal of Environmental studies and management. Vol. 6 no.3

[11] Suleman, Y., and Ifabiyi, I., (2014), "The Role of Rainfall Variability in Reservoir storage Management at Shiroro Hydropower Dam, Nigeria" An international journal of Science and technology. Vol. 3, iss. 2

[12] Ifabiyi, I. (2011b), "Contributions of Reservoir Elements to Monthly Electricity Generation in the Jebba Hydropower Reservoir, Nigeria"Ozean Journal of Applied Sciences, vol. 4, iss. 3