OPTIMIZATION OF CUTTING PARAMETERS OF TURNING PROCESS IN ENGINEERING MACHINING OPERATION, USING A GEOMETRIC PROGRAMMING APPROACH

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ABSTRACT
The production time, combined with the material removal rate, of the turning process is optimized using geometric programming approach, which is one of the non-linear programming techniques applied in the general Operations Research Study. The constraints used are the maximum cutting parameters, power and surface roughness. The approach involves theoretical modeling for production time of turning process, which is expressed as a function of the cutting parameters (cutting speed, feed rate and depth of cut). The constrained theoretical model so developed is optimized. The results of the model reveal that the method provides a systematic, easy, effective and efficient technique to obtain the optimal cutting parameters that will minimize the production time of turning process when combined with material removal rate. The rationale behind combining the production time and material removal rate is to reflect the depth of cut in the model (since depth of cut is a factor of material removal rate), and material removal rate is a factor of time which may enhance good quality of product when optimized. The model is validated using data from the turning operation of Cast iron material to produce a Counterweight (a part for Briquette Press or Briquette Machine). Scilab and MS-Excel are used to obtain the values of functions and to graphically determine the minimum production time. It is observed that higher cutting parameters lead to lower production time; but if too high or too low, lead to higher production time. In general, the model can be validated for the turning process of engineering machining operation using any metallic materials to machine a specific product, in accordance to specifications.

Keywords - Cutting Parameters, Geometric Programming, Machining Operation, Optimization, Production Time, Theoretical Model, Turning Process.

Nomenclature –
\[ P_t = \text{production time per piece (min./piece)} \]
\[ t_m = \text{time required to machine a work piece (min.)} = \frac{\pi dl}{1000vf} \]
\[ t_c = \text{tool changing time (min.)} \]
\[ t_h = \text{tool handling time (min.)} \]
\[ \text{MRR} = \text{material removal rate (in.}^3/\text{min.}) \]
\[ \alpha_1 = \text{constant} = \frac{\pi dl}{1000} \]
\[ \alpha_2 = \text{constant} = \frac{\pi dl t_c}{1000Z^3} \]
\[ \alpha_3 = \text{constant} = 12 \]
\[ v = \text{the cutting speed (m/min.)} \]
\[ f = \text{feed rate (mm/revolution)} \]
\[ D_c = \text{depth of cut (mm)} \]
\[ N = \text{spindle speed or rotational speed of the workpiece (rpm)} \]
\[ F = \text{cutting force} \]
\[ F_m = \text{the feed speed (mm/min.)} \]
\[ d = \text{average diameter of the work piece (mm.)} \]
\[ l = \text{average length of the work piece (mm.)} \]
\[ R = \text{nose radius of the tool (mm)} \]
\[ R_a = \text{average surface roughness (μm)} \]
\[ P_w = \text{power requirement (kW)} \]
\[ T = \text{tool life (min.)} \]
\[ \eta = \text{efficiency of cutting} \]
\[ p = \text{constant} \]
\[ n = \text{constant} \]
\[ Z = \text{constant} \]
\[ \pi = \text{constant} = \frac{22}{7} \]

1 INTRODUCTION

In turning process of engineering machining operation, optimum selection of cutting parameters importantly contribute to efficiency and also increase productivity. This is made possible by minimizing the production time of the process via optimization techniques. According to Deepak [6]: “Time is the most important parameter in any operation and all the manufacturing firms aim at producing a product in minimum time to reach the customer quickly and enhance the customer satisfaction”. The success of an optimization technique lies on the best time in which it provides a solution to the manufacturing firms, not necessarily in its complexity.

“Machining processes are the core of manufacturing industry, where raw material is shaped into a desired product by removing unwanted material”, Yograj and Pinkey [23]. The products of machining operations like pulleys, shafts, bearings, bushings, sleeves, bolts, nuts, screws, counterweights, are parts for producing machinery and other equipment found in manufacturing industries. The selection of optimal (best) cutting parameters forms a very important part of the turning process in engineering machining operation.

Turning defined as the machining operation that produces cylindrical parts by the CNC lathe machine, is the operation performed most commonly in industries and manufacturing firms. Turning is used to reduce the diameter of the work piece, usually to a specified dimension, and to produce a smooth finish on the metal. Often the work piece will be turned so that adjacent sections have different diameters. Or, in its basic form, it can be defined as “the machining of an external surface:

(i) With the work piece rotating
(ii) With a single-point cutting tool, and
(iii) With the cutting tool feeding parallel to the axis of the work piece and at a distance that will remove the outer surface of the work.” Umesh [22].

Researchers have been trying in the past to optimize the turning process by finding the optimal values of the turning process parameters that produce the optimum results. According to Deepak [4], “Generally, in workshops, the cutting parameters are selected from machining databases or specialized handbooks, but the values obtained from these sources are actually the starting values and not the optimal values. Optimal cutting parameters are the key to economical turning operations.”

The first necessary step for process parameter optimization in any turning process is to understand the principles governing the turning processes by developing an explicit mathematical model, which may be of three types: mechanistic, theoretical and empirical. To determine the optimal cutting parameters, reliable mathematical models have to be formulated to associate the cutting parameters with the cutting performance. However, it is also well known that reliable mathematical models are not easy to obtain. The technology of turning process has grown substantially over time owing to the contribution from many branches of engineering with a common goal of achieving higher machining process efficiency. Selection of optimal machining condition is a key factor in achieving this purpose. The maximum production rate for turning process is obtained when the total production time is minimal. Similarly, good quality of product (smooth finish) is maintained when the material removal rate is cautiously controlled and the surface roughness is taken as a constraint in finishing operations. Deepak [6] stated that: “Surface roughness can be used as a constraint in finishing operations. Therefore, it becomes a very
important factor in determining finish cutting conditions”. According to Nithyanandhan et al. [10], “In machining process, Surface finish is one of the most significant technical requirements of the customer.” It is pertinent to note that cutting parameters should be selected to optimize the economics of machining operations, as assessed by productivity, efficiency or some other suitable criteria. According to Thakre [20], productivity could be interpreted in terms of material removal rate in the machining processes. “The cutting conditions that determine the rate of metal removal are the cutting speed, the feed rate, and the depth of cut.” Oberg et al. [11].

Deepak [6] presented an analysis on cutting speed and feed rate optimization for minimizing production time of turning process. He performed optimization using only the cutting speed and feed rate as decision variables in the model, maximum cutting speed, maximum feed rate, power and surface roughness as constraints, which he analyzed independently. It was revealed that the proposed method provided a systematic and efficient method to obtain the minimum production time for turning. Also, the method of GP could be applied successfully to optimize the production time of turning process.

In this research work, we shall include the material removal rate so as to reflect the depth of cut in the model, then explore simple, effective and efficient way of optimizing the production time of the turning process within the following operating constraints - the maximum cutting speed, maximum feed rate, maximum depth of cut, power requirement and surface roughness. GP approach will be used in obtaining the optimal solution.

1.1 Scope and Purpose

This research covers the area of engineering machining operation that concerns the turning process. The cutting parameters (cutting speed, feed rate and depth of cut) form the input (design or decision) variables in the developed GP model for the minimization of production time. The decision variables in the model are optimized using the GP technique. We then apply the model to a real life data generated from metal machining operations using Cast iron material as the work piece. A sensitivity analysis is finally carried out to check the robustness of the model. This approach is intended to reduce the time it would take to produce an item of turning process by recommending the optimum values of cutting parameters, which form the decision variables in the model.

2 LITERATURE REVIEW

Attempt is made to review the literature on optimizing machining parameters in turning processes. Various conventional techniques employed for machining optimization include geometric programming, geometric plus linear programming, goal programming, sequential unconstrained minimization technique, dynamic programming, and so on.

In this research work, geometric programming approach will be used in the modeling of cutting parameters problem in machining operations. It is a powerful tool for solving some special type of non-linear programming problems. Generally, it has a wide range of applications in optimization, and particularly, in engineering for solving some complex optimization problems. According to Islam and Roy [9], since late 1960’s, GP has been known and used in various fields like Economics, Physical Sciences, Engineering, et cetera. Non-linear programming problems are perhaps the most tedious class of optimizing problems to deal with, because the response function and sometimes constraints are both non-linear and there exists no handy transformation to simplify or reduce them to linear.

Many applications of GP are on engineering design problems where parameters are estimated. “Following the pioneer work of Taylor (1907) and his famous tool life equation, different analytical and experimental approaches for the optimization of
machining parameters have been investigated”, see Roby et al. [16].

“In 1967 Duffin, Peterson and Zener put a foundation stone to solve wide range of engineering problems by developing basic theories of GP in the book Geometric Programming”, see Das and Roy [3].

Also in 1976, according to Das and Roy [3], Beightler and Phillips gave a full account of entire modern theory of GP and numerous examples of successful applications of GP to real-world problems in their book Applied Geometric Programming.

Umesh [22] did analysis on Optimization of Surface Roughness, Material Removal Rate and Cutting Tool Flank Wear in Turning Using Extended Taguchi Approach. The study dealt with optimization of multiple surface roughness parameters along with material removal rate (MRR) in search of an optimal parametric combination (favourable process environment) capable of producing desired surface quality of the turned product in a relatively lesser time (enhancement in productivity). The study proposed an integrated optimization approach using Principal Component Analysis (PCA), utility concept in combination with Taguchi’s robust design of optimization methodology. The following conclusions were drawn from the results of the experiments and analysis of the experimental data in connection with correlated multi-response optimization in turning:

1) Application of PCA was recommended to eliminate response correlation by converting correlated responses into uncorrelated quality indices called principal components which have been as treated as response variables for optimization.

2) Based on accountability proportion (AP) and cumulative accountability proportion (CAP), PCA analysis could reduce the number of response variables to be taken under consideration for optimization, which was really helpful in situations where large number of responses have to be optimized simultaneously.

3) Utility based Taguchi method was found fruitful for evaluating the optimum parameter setting and solving such a multi-objective optimization problem.

4) They asserted that the technique they adopted can be recommended for continuous quality improvement and off-line quality control of a process/product.

Ojha and Biswal [13] concluded that by using weighted method we can solve a multi-objective geometric programming problem as a vector-minimum problem. A vector-maximum problem can be transformed as a vector-minimization problem. If any of the objective function and/or constraint does not satisfy the property of a posynomial after the transformation, then any of the general purpose nonlinear programming algorithms could be used to solve the problem. This technique could also be used to solve a multi-objective signomial geometric programming problem. However, if a GP problem has either a higher degree of difficulty or a negative degree of difficulty, then any of the general purpose nonlinear programming algorithms could be used instead of a GP algorithm.

Ojha and Das [14] also carried out a research on Multi-Objective Geometric Programming Problem Being Cost Coefficients as Continuous Function with Weighted Mean Method. Their conclusion was the same as that of Ojha and Biswal [13].

Agarwal [1], after thirty-six (36) specimens of Aluminium alloy were machined, noted that the surface roughness could be efficiently calculated by using spindle speed, feed rate and depth of cut as the input variables.

Deepak [6] modeled the cutting speed and feed rate for the minimum production time of a turning operation, using GP approach. The maximum cutting speed, the maximum feed rate, maximum power available and the surface
roughness, were taken as constraints. The results of the model showed that the proposed method provided a systematic and efficient method to obtain the minimum production time for turning. It was concluded from his study that the obtained model could be used effectively to determine the optimum values of cutting speed and feed rate that will result in minimum production time. It was also revealed that the method of GP could be applied successfully to optimize the production time of turning process.

Yograj and Pinkey [23] addressed and solved the problem of parameter optimization in constrained machining environment using Differential Evolution (DE), a potential candidate of non-traditional global optimizers. The mathematical simulation turned out as a complex and highly constrained machining model where the aim was to minimize the total production cost. The machining parameters as feed rate, cutting speed and depth of cut during roughing and finishing passes were the main process parameters whose optimal values affect the machining process to greater extent. The observation of optimal results showed that the proposed methods (DE), provided promising results on quality and feasibility basis as compared to other existing algorithms. The proposed algorithm found significantly better optimal solution with less computational efforts. Thus, DE could be recommended as a reliable and efficient method for solving such complex machining problems and those with higher degree of complexity.

3 MATHEMATICAL MODELING FOR OPTIMIZATION

According to Arua et al. [2], “Modeling is very important in operations research. Though model has different shades of meaning…” Here we have a GP model formulation, which is theoretical and has a mathematical expression.

The Production time to produce a part by turning operation is denoted by \( P_t \), and expressed as follows:

\[
P_t = \text{Machining Time} + \text{Tool Changing Time} + \text{Set-up Time}
\]  

(1)

We now add the Material Removal Rate (MRR)

\[
P_t + \text{MRR} = \text{Machining Time} + \text{Tool Changing Time} + \text{Set-up Time} + \text{MRR}
\]  

(2)

Let us define:

\[
P_{\text{MRR}} = \text{production time per piece (min./piece)} + \text{material removal rate (in.}^3/\text{min.})
\]

Then we have:

\[
P_{t\text{MRR}} = \left( P_t + \text{MRR} \right) = t_m + t_c \left( \frac{t_m}{T} \right) + t_h + 12vfD_c
\]

Thus,

\[
P_{t\text{MRR}} = t_m + t_c \left( \frac{t_m}{T} \right) + t_h + 12vfD_c
\]  

(3)

Where,

\[
\text{MRR} = 12vfD_c; \ Oberg \ et \ al. \ [11]
\]

The Taylor's tool life denoted by \( T \), used in Equation (3) above is given by:

\[
T = \left( \frac{Z}{f^P \nu} \right)^\frac{1}{n}
\]  

(4)

Where, \( n \), \( p \) and \( Z \) are constants in tool life equation; depend on the many factors like tool geometry, tool material, work piece material, etc., according to Deepak [5].

In Tool-life testing, very high standards of systematic tool testing were set by F.W. Taylor in the work which culminated in the development of high speed steel. The variable of cutting speed, feed rate, depth of cut, tool geometry and lubricants, as well as tool material and heat treatment were studied and the results presented as mathematical relationships for tool life as a function of all these parameters. These tests were all carried out by lathe turning of very large steel billets using single point tools. Such elaborate tests have been too expensive.
in time and manpower to be repeated frequently, and it has become customary to use standardized conditions, with cutting speed and feed rate as the only variables. The results are presented using what is called Taylor’s equation, according to Trent [21].

This implies that:

\[
P_{\text{IMRR}} = t_m + t_c \left( \frac{t_m}{Z_{v}^{\frac{1}{n}}} \right) + t_h + 12vfD_c
\]

\[
= t_m + t_c \left( \frac{tm(f^{p} v)^{\frac{1}{n}}}{(Z_{v}^{n})^{\frac{1}{n}}} \right) + t_h + 12vfD_c
\]

where,

\[
t_m = \frac{\pi dl}{1000vf}
\]

So,

\[
P_{\text{IMRR}} = \frac{\pi dl}{1000vf} + t_c \left( \frac{\pi dl}{1000vf} \right)^{\frac{1}{n}} + t_h + 12vfD_c
\]

\[
= \left( \frac{\pi dl}{1000} \right)v^{-1}f^{-1} + \left( \frac{\pi dl t_c}{1000(Z_{v})^{\frac{1}{n}}} \right)v^{\frac{1}{n}}f^{(\frac{1}{n})^{-1}} + t_h + 12vfD_c
\]

But the tool handling time \( (t_h) \) does not depend on the Cutting speed \( (v) \), Feed rate \( (f) \) and Depth of cut \( (D_c) \); Deepak [6].

Therefore,

\[
P_{\text{IMRR}} = \left( \frac{\pi dl}{1000} \right)v^{-1}f^{-1} + \left( \frac{\pi dl t_c}{1000(Z_{v})^{\frac{1}{n}}} \right)^{\frac{1}{n}} + 12vfD_c
\]

For convenience, let us define constant coefficients:

\[
\alpha_1 = \left( \frac{\pi dl}{1000} \right)
\]

\[
\alpha_2 = \left( \frac{\pi dl t_c}{1000Z_{v}^{\frac{1}{n}}} \right)
\]

\[
\alpha_3 = 12
\]

Then, substituting we have the model, which is now a function of the cutting parameters—Cutting speed \( (v) \), Feed rate \( (f) \) and Depth of cut \( (D_c) \), as follows:

\[
P_{\text{IMRR}}(v,f,D_c) = \alpha_1 v^{-1}f^{-1} + \alpha_2 v^{\left(\frac{1-n}{n}\right)}f^{\left(\frac{p-n}{n}\right)} + \alpha_3 vfD_c
\]

(5)

It is necessary to state that a function of the form, as presented in equation (5) above can also be written as:

\[
P_{\text{IMRR}}(v,f,D_c) = \alpha_1 v^{-1}D_c^{\theta} + \alpha_2 v^{\left(\frac{1-n}{n}\right)}f^{\left(\frac{p-n}{n}\right)}D_c^{\theta} + \alpha_3 vf^{\theta}D_c^{\theta}
\]

:see Taha [19].

4 OPTIMIZATION OF MODEL USING GEOMETRIC PROGRAMMING

“The fundamental problem of optimization is to arrive at the best possible decision in any given set of circumstance,” see Arua et al. [2]. Our interest here is to optimize the cutting parameters, which form the decision variables (variables of interest or design variables) in the objective function (the model) in consideration of the associated constraints (which are combined as a set), using a GP approach.

4.1 Concept of Geometric Programming

GP is an important class of optimization problems that enable statisticians and other practitioners to model a large variety of real-world applications, mostly in the field of engineering design and operations. A GP is a type of mathematical optimization problem characterized by objective and constraint functions that have a special form. To justify the use of GP approach for
optimization of cutting parameters in this research work, it is pertinent to define such problems that GP deals with. We then consider problems in which the objective and the constraint functions are of the following type:

\[ Z = f(x) = \sum_{j=1}^{N} U_j \]

where

\[ U_j = C_j \prod_{i=1}^{j} x_i^{a_{ij}} \]

(6)

It is assumed that:

i. All \( C_j > 0, C_j, \) of course, are constants.

ii. \( N < \infty, \) that is, \( N \) is finite.

iii. The exponents, \( a_{ij} \) are unrestricted in sign.

iv. The function, \( f(x) \) takes the form of a polynomial except that the exponents, \( a_{ij} \) may be negative. For this reason, and because all \( C_j > 0, \) \( f(x) \) is called a posynomial, Taha [19]

Similarly, when we apply nonlinear programming to engineering design problems, as well as certain economics and statistics problems, the objective function and the constraint functions frequently take the form of GP (as above), Hillier and Lieberman [7]. We now justify the use of GP approach for optimization of cutting parameters by comparing our model with equations (6) and (7), which vividly, we can discover that the approach is satisfied.

In relation to the above definition of GP, therefore we proceed as follows:

\[ \text{Minimize (Min.) } p_{\text{ess}}(v_f, D_c) = \alpha_1 v_f^{-1} f^{-1} + \alpha_2 v_f^2 \]

(7)

\[ + \alpha_3 v_f D_c \]

(A)

Subject to the following constraints:

- Maximum Cutting parameters (Maximum Cutting speed, Maximum Feed rate, Maximum Depth of cut), Power and Surface roughness constraints

The constraints above imply the following condition:

\[ v_f D_c P_w R_a \leq v_{\text{max}} f_{\text{max}} D_{\text{c max}} P_{w \text{ max}} R_{a \text{ max}} \]

where,

\[ P_w = \frac{(F \times v_f)}{6120 \eta} \]

\[ R_a = \frac{f^2}{32R} \]

\( v_{\text{max}} f_{\text{max}} D_{\text{c max}} \) are the maximum cutting parameters allowable on the lathe, \( P_{w \text{ max}} \) and \( R_{a \text{ max}} \) are the maximum power required and maximum surface roughness respectively, allowable for the turning operation.

Substituting, inequality above implies that:

\[ \left( \frac{F v_f^2 f_3 D_c}{6120 \eta \times 32R} \right) \leq 1 \]

(8)

\[ \left( \frac{v_{\text{max}} f_{\text{max}} D_{\text{c max}} P_{w \text{ max}} R_{a \text{ max}}}{6120 \eta \times 32R} \right) \]

which now yields:
Let us define,

\[
\alpha_4 = \left( \frac{F v^2 f^3 D_c}{195840 \eta R (v_{\text{max}} f_{\text{max}} D_{c_{\text{max}}} P_{w_{\text{max}}} R_{a_{\text{max}}})} \right)
\]

then we conclude that:

\[
\alpha_4 (v^2 f^3 D_c) \leq 1
\]  

(8)

Now, to enable us perform optimization, we apply the conclusion from the above derivations.

Thus we have:

\[
\min \quad P_{\text{BES}}(v, f, D_c) = \alpha_1 v^{-2} f^{-1} + \alpha_2 v^{\left(\frac{-2}{a} + \frac{8}{a}\right)} + \alpha_3 v f D_c \quad \text{subject to} \quad \alpha_4 (v^2 f^3 D_c) \leq 1
\]

(B)

v \geq 0, f \geq 0, D_c \geq 0 \quad \text{(the non-negativity constraints)}; \quad \text{while } \alpha_1, \alpha_2, \alpha_3 \text{ and } \alpha_4 \text{ are machining constants.}

We now wish to formulate this as a Zero-degree-of-difficulty problem, considering the number of functions in the objective function and the constraints, and also the number of decision variables in the objective function. “The degree of difficulty is defined as the number of terms minus the number of variables minus one, and is equal to the dimension of the dual problem”, Ojha and Biswal [13].

Simply, we have “the degree of difficulty” as follows:

Degree of difficulty = (N–n–1)

Where,

N = the number of functions in the objective function and the constraints (that is, the total number of posynomial terms in the problem), which is equal to four (4).

n = the number of decision variables (cutting parameters) in the objective function, which is equal to three (3).

Therefore, the degree-of-difficulty becomes (4–3–1) = 0, which now justifies the desired “Zero-degree-of-difficulty” for this problem.

4.2 Constrained Minimization

When an objective function is to be minimized, it is workable if either the minimization problem is constrained (or unconstrained) depending on the problem formulation and its environment. We discover that most engineering optimization problems are subject to constraints. If the objective function and all the constraints are expressible in the form of posynomials, GP can be used most conveniently to solve the optimization problem: see Rao [15]. GP problem whose parameters, except for exponents, are all positive are called posynomial problems, whereas GP problems with some negative parameters are referred to as signomial problems: see Ojha and Biswal [13].

Let us consider the constrained minimization problem whereby we are required to find the design variables, \( X = x_i, \quad i = 1, 2, \ldots, n \), which minimize the objective function:

\[
Z = f(x) = \sum_{j=1}^{N_0} c_{0j} \prod_{i=1}^{n} x_i^{a_{ij}}
\]  

Subject to:

\[
g_k(x) = \sum_{j=1}^{N_k} c_{kj} \prod_{i=1}^{n} x_i^{a_{ij}} \leq 1, \quad k = 1, 2, \ldots, m
\]

(9)

Where,
$C_{0j} \ (j = 1, 2, \ldots, N_0)$ and $C_{kj} \ (k = 1, 2, \ldots, m; \ j = 1, 2, \ldots, N_k)$ are the coefficients, which are positive numbers. $a_{0ij} \ (i = 1, 2, \ldots, n; \ j = 1, 2, \ldots, N_0)$ and $a_{kij} \ (k = 1, 2, \ldots, m; \ j = 1, 2, \ldots, N_k)$ are any real numbers. $m$ indicates the total number of constraints, $N_0$ represents the number of terms in the objective function, and $N_k$ denotes the number of terms in the $k^{th}$ constraint. The design variables, $X = x_i, \ i = 1, 2, \ldots, n$, are expected to assume only positive values in equations (9) and (10). Now, considering the attributes possessed, equations (9) and (10) above are posynomial functions: see also Taha [19].

4.3 Combination of Maximum Cutting Parameters, Power and Surface Roughness as Constraints for Optimization of Cutting Parameters

For primal and dual programs in the case of less-than inequalities, if the original problem has a zero degree of difficulty, the minimum of the primal problem can be obtained by maximizing the corresponding dual function, Rao [15].

Let us now consider the primal as follows:

$$\text{Minimize (Min.)} \quad P^*_{\text{PES}}(v,f,D_c) = a_0 v^{-1} f^{-1} + a_1 v^{-1} \left( \frac{f}{a} \right)^{1/3} + a_2 v D_c$$

Subject to: $\alpha_4 (v^2 f^3 D_c) \leq 1$

We then have the corresponding dual functions as:

$$Z^* = P^*(x) = P^*_{\text{MAX}}(v,f,D_c) = \prod_{j=1}^{n} \left[ \frac{a_j}{w_j} \right]^{w_j} \prod_{i=1}^{m} \left[ \sum_{r=1}^{p_i} \frac{a_{ij}}{w_{ij}} \right]^{w_{ij}} \prod_{i=1}^{m} \left( V_i \right)^{V}$$

(11)

where, $V = \sum_{r=1}^{P_i} w_{ir}$

Equation (11) is transformed to:

Maximize (Max.)

$$P^*_{\text{MAX}}(v,f,D_c) = \left( \frac{a_1}{w_1} \right)^{w_1} \left( \frac{a_2}{w_2} \right)^{w_2} \left( \frac{a_3}{w_3} \right)^{w_3} \left( \frac{a_4}{w_4} \right)^{w_4}$$

(12)

which can be reduced to:

Maximize (Max.)

$$P^*_{\text{MAX}}(v,f,D_c) = \left( \frac{a_1}{w_1} \right)^{w_1} \left( \frac{a_2}{w_2} \right)^{w_2} \left( \frac{a_3}{w_3} \right)^{w_3} (a_4)$$

(13)

Where,

$w_1, w_2, w_3$ and $w_4$ are the corresponding normalizing weights, which are also referred to as the dual variables, Rao [15].

Equations (11) and (13) are subject to the Normality and the Orthogonality conditions as follows:

$$w_1 + w_2 + w_3 = 1$$

(14)

$$-w_1 + \left( \frac{1-n}{n} \right) w_2 + w_3 + 2w_4 = 0$$

(15)

$$-w_1 + \left( \frac{p-n}{n} \right) w_2 + w_3 + 3w_4 = 0$$

(16)

$$w_3 + w_4 = 0$$

(17)

The simultaneous equations (14), (15), (16) and (17) above, which are the Normality and the Orthogonality conditions, can be put in a matrix form:

$$AW = b$$

(18)

Where,

$A$ is a $(4 x 4)$ square matrix, $W$ is a $(4 x 1)$ column vector and $b$ is a column vector, or a $(4 x 1)$ identity matrix corresponding to $A$ and $W$: Taha [19].

Then, we have the arrangement as follows:

$\text{Maximize (Max.)} \quad P^*_{\text{MAX}}(v,f,D_c) = \left( \frac{a_1}{w_1} \right)^{w_1} \left( \frac{a_2}{w_2} \right)^{w_2} \left( \frac{a_3}{w_3} \right)^{w_3} (a_4)$$

(13)
For clarity, equations (18) and (19) above can be compared. So, the equations are solved simultaneously for the dual variables in the following manner:

Equations (15), (16) and (17) can be written as:

\[
\begin{align*}
\frac{1}{n}w_1 &= \left(\frac{1-n}{n}\right) w_2 + w_3 + 2w_4 \\
\frac{p-n}{n}w_1 &= \left(\frac{p-n}{n}\right) w_2 + w_3 + 3w_4 \\
-w_3 &= w_4
\end{align*}
\]

(20)

(21)

(22)

Equating equations (20) and (21), then solving simultaneously, we have:

\[
\begin{align*}
\left(\frac{1-n}{n}\right) w_2 + w_3 + 2w_4 &= \left(\frac{p-n}{n}\right) w_2 + w_3 + 3w_4 \\
\left(\frac{1-n}{n}\right) w_2 - \left(\frac{p-n}{n}\right) w_2 &= 3w_4 - 2w_4 \\
\left(\frac{1-n-p+n}{n}\right) w_2 &= w_4
\end{align*}
\]

(23)

(24)

(25)

From equation (22), we discover that:

\[
\left(\frac{2-n-p}{n}\right) w_2 + w_2 + \left(\frac{p-1}{n}\right) w_2 = 1
\]

(26)

Substituting equations (26), (27) and (28) in equation (22),

\[
\left(\frac{2-n-p}{n}\right) w_2 + w_2 + \left(\frac{p-1}{n}\right) w_2 = 1
\]

(27)

Substituting the values of the normalizing weights appropriately, in equation (13) above we have:

\[
\text{Maximize } P^*_{\text{Max}}(v_i;D_c) = \left(\frac{q_i}{(2-n-p)}\right) ^{\frac{1}{(2-n-p)}} \left(\frac{q_i}{n}\right) ^{\frac{1}{n}} \left(\frac{q_i}{(p-1)}\right) ^{\frac{1}{(p-1)}} \left(q_i\right)^{\frac{1}{(p-1)}}
\]

(34)

Now, we optimize the cutting parameters; the cutting speed (v), the feed rate (f) and Depth of cut (Dc), using the following conditions:
From equation (34),
\[ f^{-1} = \frac{w_1 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_1 v^{-1}} \]
which implies that:
\[ f = \frac{\alpha_1 v^{-1}}{w_1 \times P_t \cdot \text{MRR}(v,f,D_c)} \]  
(38)

From equation (35),
\[ f^{(\frac{p-n}{n})} = \frac{w_2 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_2 v^{-\left(\frac{1-n}{n}\right)}} \]
which implies that:
\[ f = \left(\frac{w_2 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_3 v^{-\left(\frac{1-n}{n}\right)}}\right)^{\left(\frac{n}{p-n}\right)} \]  
(39)

Equating equations (38) and (39), then solving for \( v \), therefore:
\[ v = \left(\frac{w_1 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_1}\right)^{\left(\frac{p-n}{1-p}\right)} \left(\frac{w_2 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_2}\right)^{\left(\frac{n}{1-p}\right)} \]  
(40)

Substituting equation (40) in either equation (38) or equation (39), and solving for \( f \), therefore:
\[ f = \left(\frac{\alpha_1}{w_1 \times P_t \cdot \text{MRR}(v,f,D_c)}\right)^{\left(\frac{1-n}{1-p}\right)} \left(\frac{\alpha_2}{w_2 \times P_t \cdot \text{MRR}(v,f,D_c)}\right)^{\left(\frac{n}{1-p}\right)} \]  
(41)

Also, from equation (37),
\[ D_c = \frac{w_3 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_3 vf} \]  
(42)

Substituting equations (40) and (41) in equation (42), therefore:
\[ D_c = \frac{w_3 \times P_t \cdot \text{MRR}(v,f,D_c)}{(\alpha_3 \left(\frac{w_1 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_1}\right)^{\left(\frac{p-n}{1-p}\right)}} {\left(\frac{n}{1-p}\right)} \left(\frac{w_1 \times P_t \cdot \text{MRR}(v,f,D_c)}{\alpha_1}\right)^{\left(\frac{n}{1-p}\right)}} \]  
(43)

For convenience in application, however, equation (42) can be used once equations (40) and (41) are derived. This also implies that from equation (42), any of the cutting parameters can be made the subject which is dependent on the others, or simply put, the dependent variable.

In summary, the cutting parameters, the cutting speed (\( v \)), the feed rate (\( f \)) and the depth of cut (\( D_c \)) have been optimized for a set of constraints combined together, to yield the desired positive result. Thus, when different values of the machining constants and other relevant parameters for turning operation are provided (depending on the metallic material used as work piece, and its dimension), the equations above will enable us to obtain the values of the cutting parameters; the values of the cutting parameters are tested for minimum production time and material removal rate, using the model in either equation (11) or equation (12) above.

Also, for convenience, if data for corresponding estimate values of the cutting parameters are made available via engineering machining handbooks or by mere estimation during turning process (that is, varying the cutting parameters), equation (11) or equation (12) above is the solution for deriving the minimum production time with material removal rate of any type of metallic material used as work-piece. In this instance, it is now easy to single out the cutting parameters that correspond to the minimum production time (the optimum values).
5 NUMERICAL DEMONSTRATION OF RESULTS

5.1 Machining Data for Counterweight (A Part for Briquette Press)

\[ F = 99015 \quad l = 138 \text{ mm} \]
\[ \eta = 0.85 \quad D_{c\text{ max}} = 1.00 \]
\[ F_m = 3.5 \quad p = 1.5 \]
\[ Z = 10 \quad P_{w\text{ max}} = 0.000455 \text{ kW} \]
\[ N = 1.5 \text{ rpm} \quad R = 1.2 \text{ mm} \]
\[ \pi = 2/7 \quad R_{a\text{ max}} = 10 \mu\text{m}. \]
\[ d = 103 \text{ mm.} \quad R_a = 10 \mu\text{m} \]
\[ v_{\text{max}} = 750 \quad n = 0.25 \]
\[ t_c = 0.5 \text{ min} \quad f_{\text{max}} = 0.09 \]

5.2 Experimental Data

<table>
<thead>
<tr>
<th>Runs</th>
<th>Cutting Speed ((v)) (m/min.)</th>
<th>Feed Rate ((f)) ((\text{mm/rev.}))</th>
<th>Depth of Cut ((D_c)) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.0029</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>0.0039</td>
<td>0.035</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>0.0049</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>0.0059</td>
<td>0.065</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>0.0069</td>
<td>0.08</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
<td>0.0079</td>
<td>0.095</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>0.0089</td>
<td>0.11</td>
</tr>
<tr>
<td>8</td>
<td>115</td>
<td>0.0099</td>
<td>0.125</td>
</tr>
<tr>
<td>9</td>
<td>130</td>
<td>0.0109</td>
<td>0.14</td>
</tr>
<tr>
<td>10</td>
<td>145</td>
<td>0.0119</td>
<td>0.155</td>
</tr>
<tr>
<td>11</td>
<td>160</td>
<td>0.0129</td>
<td>0.17</td>
</tr>
<tr>
<td>12</td>
<td>175</td>
<td>0.0139</td>
<td>0.185</td>
</tr>
<tr>
<td>13</td>
<td>190</td>
<td>0.0149</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Source: Machine Shop Unit of Scientific Equipment Development Institute (SEDI), Enugu, Enugu State, Nigeria
5.3 Results and Discussion

The following results were obtained:

1. A GP model (a function of the cutting parameters or the decision variables) that minimizes the production time, which is given as:

\[
P_{\text{MRR}}(v, f, D_c) = \left( \frac{n \pi l}{1000} \right) v^{-1} f^{-1} + \left( \frac{n \pi l}{1000} \right) f^{-1} + \alpha_1 v^{-1} f^{-1} + \alpha_2 \left( \frac{1 - n}{n} \right) f^{-1} + \alpha_3 v f D_c
\]

Values of constant coefficients in the model:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Constant</th>
<th>Formulae (using parameters)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \alpha_1 ) constant</td>
<td>( \frac{n \pi l}{1000} )</td>
<td>44.672571</td>
</tr>
<tr>
<td>2.</td>
<td>( \alpha_2 ) constant</td>
<td>( \frac{n \pi l}{1000Z_n} )</td>
<td>0.0022336</td>
</tr>
<tr>
<td>3.</td>
<td>( \alpha_3 ) constant</td>
<td>( \frac{1 - n}{n} )</td>
<td>12</td>
</tr>
</tbody>
</table>

2. The optimum values of cutting parameters that result in minimum production time. That is, the optimum cutting parameters that correspond to minimum production time as shown below:

\[
v = \left( \frac{w_1 x P_{\text{MRR}}^*(v, f, D_c)}{a_1} \right) \left( \frac{n - p}{1 - p} \right) \left( \frac{w_2 x P_{\text{MRR}}^*(v, f, D_c)}{a_2} \right)^{\frac{n}{n - p}}
\]

\[
f = \left( \frac{\alpha_1}{w_1 x P_{\text{MRR}}^*(v, f, D_c)} \right) \left( \frac{n - p}{1 - p} \right) \left( \frac{\alpha_2}{w_2 x P_{\text{MRR}}^*(v, f, D_c)} \right)^{\frac{n}{n - p}}
\]

\[
D_c = \left( \frac{w_3 x P_{\text{MRR}}^*(v, f, D_c)}{a_1} \right) \left( \frac{n - p}{1 - p} \right) \left( \frac{w_1 x P_{\text{MRR}}^*(v, f, D_c)}{a_2} \right)^{\frac{n}{n - p}}
\]

3. The numerical values of the optimum cutting parameters that result in minimum production time when we apply a real life data generated from metal machining operations, that is machining of Counterweight (a part for Briquette press), as shown in the table below:

<table>
<thead>
<tr>
<th>Optimum Cutting Parameters</th>
<th>Production time (min./piece)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting Speed (v) (m/min.)</td>
<td>Feed Rate (f) (mm/rev.)</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>495.35714</td>
<td>0.0002357</td>
</tr>
</tbody>
</table>

4. Graphical presentation from sensitivity analysis for checking the robustness of the model by varying the cutting parameters, with a wide range of data, as they are plotted against production time as follows:

<table>
<thead>
<tr>
<th>Runs</th>
<th>Cutting Speed (v) (m/min.)</th>
<th>Feed Rate (f) (mm/rev.)</th>
<th>Depth of Cut (Dc) (mm)</th>
<th>P_{\text{MRR}}(v, f, Dc) (min./piece)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.0029</td>
<td>0.02</td>
<td>1,540.44</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>0.0039</td>
<td>0.035</td>
<td>458.22</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>4</td>
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<td>0.065</td>
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<tr>
<td>5</td>
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<td>0.0069</td>
<td>0.08</td>
<td>92.95</td>
</tr>
<tr>
<td>6</td>
<td>85</td>
<td>0.0079</td>
<td>0.095</td>
<td>67.29</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>0.0089</td>
<td>0.11</td>
<td>51.37</td>
</tr>
<tr>
<td>8</td>
<td>115</td>
<td>0.0099</td>
<td>0.125</td>
<td>40.95</td>
</tr>
<tr>
<td>9</td>
<td>130</td>
<td>0.0109</td>
<td>0.14</td>
<td>33.91</td>
</tr>
<tr>
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<td>0.0119</td>
<td>0.155</td>
<td>29.1</td>
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<td>11</td>
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<td>0.0129</td>
<td>0.17</td>
<td>25.85</td>
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<tr>
<td>12</td>
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<td>0.0139</td>
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<tr>
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<tr>
<td>15</td>
<td>220</td>
<td>0.0169</td>
<td>0.23</td>
<td>22.28</td>
</tr>
</tbody>
</table>
The curve obtained between production time with material removal rate and cutting speed, “Fig. 1” above reveals that a smaller value of cutting speed results in a high production time. It is due to the fact that a smaller cutting speed increases the production time of parts. Also, it will decrease the profit rate due to the production of a lesser number of parts. In the same premise, if the cutting speed is too high, it will also lead to a high production time due to excessive tool wear and increased machine downtime (time during which work or production is stopped). The optimum cutting speed is somewhere in-between “too slow” and “too fast” which will yield the minimum production time and maximum production rate at the same cutting speed.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>16</td>
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<td>36</td>
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<tr>
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<tr>
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<td>745</td>
<td>0.0519</td>
<td>0.755</td>
<td>351.81</td>
</tr>
</tbody>
</table>
Also, the curve between the production time with material removal rate and the feed rate, “Fig. 2” above indicates that a smaller feed rate will result in high production time. A smaller feed rate means the number of revolutions should be increased. The more the number of revolutions, the less will be the production time. However, a very high feed rate is not advisable as it will increase the tool wear and surface roughness leading to increased machining time and machine downtime, which will result to high production time. So, the optimum feed rate is somewhere in-between “too small” and “too high” which will result in the minimum production time and maximum production rate at the same feed rate.

Again, the curve obtained between production time with material removal rate and depth of cut, “Fig. 3” above depicts that a smaller value of depth of cut results in a high production time. It is due to the fact that a smaller depth of cut increases the production time of parts. Also, it will decrease the profit rate due to the production of a lesser number of parts. Similarly, if the depth of cut is too high, it will also lead to a high production time due to excessive tool wear and increased machine downtime. The optimum depth of cut is somewhere in-between “too low” and “too high” which will yield the minimum production time, maximum production rate and better quality of machined part.
6 CONCLUSION

In this work, the cutting speed, feed rate and depth of cut are modeled (applying formulas for production time and material removal rate, and at the same time, considering the Taylor's tool life equation) in search of an optimal parametric combination for the minimum production time of a turning operation using GP approach. The maximum cutting parameters (maximum cutting speed, maximum feed rate and maximum depth of cut), the maximum power available and the surface roughness are taken as constraints. The results of the model show that the proposed method provides a systematic and efficient method to obtain the minimum production time with material removal rate for turning. This approach helps in quick analysis of the optimal region which will yield a small production time and smooth finish of machined part rather than focusing too much on a particular point of optimization. It saves a lot of time and can be easily implemented by manufacturing firms. The coefficients n, p and Z of the extended Taylor's tool life equation are not described in depth for all cutting tool and work piece combinations. Obtaining these coefficients experimentally requires lot of time, resources and then, the analysis of the obtained values increases the complexity of the process. It can be concluded from this study that the obtained model can be used effectively to determine the optimum values of cutting speed, feed rate and depth of cut that will result in minimum production time and good quality of product. The developed model saves a considerable time in obtaining the optimum values of the cutting parameters.

REFERENCES


